The Discriminant Investigation

Purpose: to discover the three possible solution types for a quadratic
to discover a link in the value of the discriminant and the nature of the roots
to discover the parameters for a, b, and c that indicate the various roots
to discover how the roots are related to the position of the graph with respect to the x–axis

Background: previous knowledge of the quadratic formula
previous practice graphing quadratics in both standard and vertex form
previous knowledge of complex (or imaginary) numbers

Investigation:

1. What are the three possible types of roots yielded by the quadratic formula? Use the quadratic formula to solve each of the following. *Categorize each solution with respect to being real or complex (imaginary), equal or distinct.*

   a) \( \frac{15}{2}x^2 - \frac{11}{2}x - 7 = 0 \)

   (4)

   b) \( 25x^2 - 30x + 9 = 0 \)

   (4)
2. What is the connection between the value of the discriminant and the type of roots yielded in question #1?

3. Graph each of the quadratics from question #1. You must complete the square to show the vertex form for each, find the x and y-intercepts, and scale the graphs accordingly. (4 marks each)

a) \[ y = \frac{15}{2} x^2 - \frac{11}{2} x - 7 \]
b) \[ y = 25x^2 - 30x + 9 \]

c) \[ y = \frac{-x^2}{3} + x - 1 \]
4. Discuss the graph orientation for each with respect to the x-axis. Use words that include reference to touching or crossing the x-axis.

a) \( y = 25x^2 - 30x + 9 \) 

(2)

b) \( y = \frac{-x^2}{3} + x - 1 \) 

(2)

c) \( y = \frac{15}{2}x^2 - \frac{11}{2}x - 7 \) 

(2)

5. Discuss the link between the nature of the roots, the orientation of the graph and the value of the discriminant. When referencing the value of the discriminant, use generalizations with respect to zero. Include ALL THREE ASPECTS when pulling your conclusions together.
Apply Your Conclusions:

1. For each of the following, USE THE DISCRIMINANT to discuss the nature of the roots and the orientation of the graph.

   a) $y = 5x^2 - 8x - 1$

   (4)

   b) $y = 2x^2 + 3x + 3$

   (4)

   c) $y = 9x^2 + 12x + 4$

   (4)