LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson : 3 - 3**

Unit/Chapter: Exponents & Logarithms **Topic: Exponential Functions**

- ***homework check:*** **FM11** exercise 1.12 p. 25 #3, 4 and
  AW11 exercise 1.6 p. 51 #12

- ***note:*** **Exponential Functions**

Recall the basic exponential function $y = b^x$, $b > 0$, and $x \in \Re$. In general, there are two types of exponential functions, one representing exponential growth (continually increasing) and one representing exponential decay (continually decreasing). Also recall, that the exponential functions follow the same rules of transformations as polynomial functions. Therefore, the variables in the transformation of $y = a \cdot b^{x+g} + h$ have the same effects: the "a" defines a vertical stretch or compression, "the g" defines a horizontal translation, and the "h" defines a vertical translation. Reflections in either axis can also be part of these transformations.

The graphs of exponential growth or decay can also be used to determine roots of exponential equations through interpolation (Note: only for those equations that cannot be solved using regular techniques).

examples) Using the graphs provided, estimate the roots of the given equations.

$2.5 = 2^x$	$3.2 = 3^x$	$5 = 10^x$
$x = 1.3$	$x = 1.05$	$x = 0.7$

- ***homework assignment:*** **FM12** p. 219 # 1 – 7
  ASM12 p. 368 # 6 – 12

(a) $y = 2^{2.3}$
(b) $2^x = 5.7$
(c) $y = 2^{6.7}$
(d) $2^x = 28$

SOLUTION:

(a) From the graph $y \doteq 4.9$
(b) From the graph $x \doteq 2.5$
(c) $2^{6.7} = 2^4 \times 2^{2.7}$
$= 16 \times 2^{2.7}$
$\doteq 16(6.5)$
$= 104.0$
(d) $2^x = 4 \times 7$
$= 2^2 \times 7$
$\doteq 2^2 \times 2^{2.8}$
$= 2^{4.8}$

$\therefore x = 4.8$

EXERCISE 7.1

A 1. Use the given graphs to find approximate values of the following numbers.

(a) $2^{1.7}$ (b) $2^{2.2}$ (c) $3^{1.9}$ (d) $5^{1.3}$
(e) $10^{0.7}$ (f) $2^{-0.6}$ (g) $5^{0.2}$ (h) $3^{-1.4}$
(i) $\sqrt{10}$ (j) $3^{1.5}$ (k) $10^{-0.4}$ (l) $2^{3.1}$

2. Use the given graphs to find an approximate value of x satisfying the following equations.

(a) $2^x = 6$ (b) $2^x = 1.8$ (c) $2^x = 0.5$
(d) $3^x = 10$ (e) $3^x = 2$ (f) $5^x = 8$
(g) $5^x = 0.8$ (h) $10^x = 3$ (i) $10^x = 7$
(j) $10^x = 0.1$ (k) $3^x = 1.5$ (l) $5^x = 3$

3. Use the $\boxed{y^x}$ key on your calculator to find the following quantities correct to 4 decimal places.

(a) $2^{1.3}$ (b) $10^{0.6}$ (c) $5^{\frac{4}{7}}$ (d) $3^{\frac{3}{11}}$
(e) $15^{1.73}$ (f) $96^{0.54}$ (g) $7^{-2.45}$ (h) $24^{-0.16}$
(i) 5^{π} (j) $3^{\sqrt{2}}$ (k) $\sqrt[6]{385}$ (l) $\sqrt[10]{87}$

B 4. Use the $\boxed{y^x}$ key on your calculator to draw the graphs of the following functions.

(a) $y = 4^x$
(b) $y = (\frac{1}{4})^x$
(c) What is the relationship between these graphs?

5. Use the graph of $y = 2^x$ to find approximate values of x and y in the following equations.

(a) $y = 2^{1.9}$ (b) $y = 2^{5.8}$
(c) $y = 2^{\sqrt{2}}$ (d) $y = 2^{9.7}$
(e) $2^x = 0.8$ (f) $2^x = 24$
(g) $2^x = 30$ (h) $2^x = 96$

6. Use the graphs given in this section to find approximate values of x and y in the following equations.

(a) $y = 3^{1.8}$ (b) $y = 3^{5.6}$
(c) $y = 5^{3.1}$ (d) $y = 10^{2.8}$
(e) $5^x = 7$ (f) $5^x = 50$
(g) $3^x = 36$ (h) $10^x = 90$

C 7. Graph $y = 2^{|x|}$.

MIND BENDER

A rope hangs over the side of the ship and is submerged 2 m. If the tide rises 0.6 m/h, how much of the rope is submerged after 4 h?

5. (a) $\frac{\ }{16}$ (b) 729 (c) 27 (d) 32

(e) $\frac{1}{7}$ (f) 3 (g) 1 000 000 000 (h) 2

(i) 3 (j) $\frac{11}{5}$ (k) $\frac{3}{2}$ (l) $\frac{9}{16}$

(m) $\frac{6}{7}$ (n) 2.7 (o) $\frac{1}{256}$ (p) $\frac{1}{32}$

6. (a) $\frac{5}{x^2y^3}$ (b) $\frac{64}{x^3}$ (c) $\frac{3y}{4x^2}$ (d) $\frac{b^4}{a}$

(e) $\frac{81b^8}{a^4c^{16}}$ (f) $x^7 + x^5 + x^3$ (g) $a^2 - \frac{1}{a^2}$ (h) $\frac{ab}{b^2 - a^2}$

7. (a) $x = -\frac{1}{3}$ (b) $x = 6$ (c) $x = \frac{7}{3}$ (d) 9

THE QUADRATIC FORMULA

1. (a) $x = \frac{3 \pm \sqrt{5}}{2}$ (b) $x = -3 \pm 2\sqrt{3}$

(c) $x = \frac{1 \pm \sqrt{17}}{4}$ (d) $t = -\frac{1}{3}$ or $t = -2$

(e) $s = \frac{-1 \pm \sqrt{5}}{2}$ (f) $w = \frac{-1 \pm \sqrt{3}i}{2}$

(g) $x = 1 \pm \frac{\sqrt{5}}{5}$ (h) $v = \frac{1 \pm \sqrt{7}}{3}$

(i) $x = 1 \pm i$ (j) $x = \frac{1 \pm \sqrt{2}i}{3}$

COMPOUND INTEREST

1. (a) \$2720.97 (b) \$2737.13 2. \$1685.40
3. (a) \$6564.66 (b) \$6605.32 (c) \$6626.69 (d) \$6641.35

EXERCISE 7.1

1. (a) 3.2 (b) 4.6 (c) 8.1 (d) 8.1 (e) 5 (f) 0.7
(g) 1.4 (h) 0.2 (i) 3.2 (j) 5.2 (k) 0.4 (l) 8.6
2. (a) 2.6 (b) 0.85 (c) −1 (d) 2.1 (e) 0.6 (f) 1.3
(g) −0.14 (h) 0.5 (i) 0.85 (j) −1 (k) 0.4 (l) 0.7
3. (a) 2.4622 (b) 3.9810 (c) 2.5084 (d) 1.3493
(e) 108.3023 (f) 11.7605 (g) 0.0085 (h) 0.6014
(i) 156.9925 (j) 4.7288 (k) 2.6971 (l) 4.2870
4. (c) The graphs are reflected in the y-axis.
5. (a) 3.7 (b) 55.7 (c) 2.7 (d) 831.7
(e) −0.3 (f) 4.6 (g) 4.9 (h) 6.6
6. (a) 7.2 (b) 469.8 (c) 146.8 (d) 631
(e) 1.2 (f) 2.4 (g) 3.3 (h) 1.95

EXERCISE 7.2

1. (a) 25 a (b) 50 a (c) 100 a (d) 75 a
2. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{64}$
3. (a) 10 000 (b) 40 000 (c) 80 000 (d) 320 000
4. (a) 12 000 (b) 24 000 (c) 48 000 (d) 96 000
5. (a) $(5 \times 10^6)2^{\frac{21}{3}}$ (b) 3.2×10^8 (c) 3.2768×10^{11} (d) 12 599 210
6. (a) 1250 (b) 320 000 (c) 5 120 000 7. $\frac{3}{4}$ h 8. 12 min
9. 72 billion

In general, the exponential function is defined by the equation,

$$y = a^x, \quad a > 0 \text{ and } x \in R,$$
$$\text{or} \quad f(x) = a^x, \quad a > 0 \text{ and } x \in R.$$

9.3 Exercise

A Use the graphs for $y = 2^x$, $y = 3^x$ and $y = 10^x$ as needed.

1 From the graph of $y = 2^x$; find the value of each of the following to one decimal place.
(a) $2^{1.3}$ (b) $2^{4.5}$ (c) $2^{3.8}$ (d) $2^{2.7}$ (e) $2^{3.3}$ (f) $2^{5.1}$

2 From the graph of $y = 3^x$, $x \in R$ find the value to 1 decimal place.
(a) $3^{2.1}$ (b) $3^{3.6}$ (c) $3^{1.5}$

3 From the graph of $y = 10^x$, $x \in R$, approximate each value to 1 decimal place.
(a) $10^{1.3}$ (b) $10^{1.8}$ (c) $10^{1.5}$

4 Solve each equation. Express the value of x to 1 decimal place.
(a) $2^x = 6$ (b) $3^x = 7$ (c) $10^x = 20$ (d) $10^x = 49$ (e) $2^x = 10$

5 Use the graph of $y = 10^x$, $x \in R$. Find the missing value. Express your answer to 1 decimal place.
(a) (1.2, ?) (b) (?, 50) (c) (1.5, ?) (d) (?, 60) (e) (1.7, ?)

B The skills for sketching graphs that you learned earlier are applied to the graphs of exponential functions.

6 (a) Construct a graph of $y = 4^x$, $x \in R$.
(b) Why is the graph in part (a) the graph of an exponential function?
(c) List the properties of the graph in part (a).

7 Use the same set of axes. Construct the graphs defined by
$y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$.
(a) How are the graphs alike? (b) How do the graphs differ?
(c) What transformation mapping of $y = 2^x$ will give $y = \left(\frac{1}{2}\right)^x$ as its image?

8 Use the same set of axes. Construct the graphs defined by $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$.

(a) How are the graphs alike? (b) How do the graphs differ?

(c) What transformation mapping of $y = 3^x$ will give $y = \left(\frac{1}{3}\right)^x$ as its image?

9 (a) Construct a graph of $y = 2^{-x}$.

(b) Did you construct the graph of a function?

(c) List the properties of the graph in part (a).

10 (a) On the same set of axes construct a graph of A: $y = 3^{-x}$ B: $y = 4^{-x}$.

(b) What properties do the graphs in part (a) have in common?

(c) How do the graphs in part (a) differ?

11 (a) Why is the term *curves of growth* appropriate to describe the graphs of $y = 2^x$, $y = 3^x$, and so on?

(b) Why is the term *curves of decay* appropriate to describe the graphs of $y = 2^{-x}$, $y = 3^{-x}$, and so on?

12 (a) Construct the graphs of

A $\boxed{y = \left(\frac{1}{2}\right)(2^x)}$ and B $\boxed{y = 2^{x-1}}$

(b) Show algebraically why the graphs of A and B are the same.

(c) Construct the graphs of

C $\boxed{y = \left(\frac{1}{3}\right)(3^x)}$ and D $\boxed{y = 3^{x-1}}$

(d) Show algebraically why the graphs of C and D are the same.

13 Use the graph of $y = 2^x$, $x \in R$ as a basis. On the same set of axes, sketch graphs of each of the following.

(a) $y = 2^x + 2$ (b) $y = 2^x - 3$ (c) $y = 2^{x-2}$

(d) $y = 2^{x+2}$ (e) $y = 2^{x+1} + 3$ (f) $y = 2^{x-2} - 3$

14 Use the graph of $y = 3^x$, $x \in R$ as a basis. On the same set of axes sketch the graph of each of the following.

(a) $y = 3^x - 2$ (b) $y = 3^{x-2}$ (c) $y = (2)(3^x)$ (d) $y = (2)(3^x) + 2$

C 15 Sketch the graph of each of the following.

(a) $y = (3)(2^{x-2}) - 1$ (b) $y = (2)(3^{x-3}) + 4$

— 24 b)x = 10 c)x = $2\sqrt{21}$ 3.48 cm 4.20 cm
cm 6.5 cm

xercise, page 329
° = 140° b)$x°$ = 45°, $y°$ = 30° c)$x°$ = 90°
= 40°, $y°$ = 110° **4.a)**40° b)100° c)90° **5.a)**25°
0° **6.a)**120° b)50° c)35° d)25°
70°, $y°$ = 20° b)$x°$ = 40°, $y°$ = 40°
= 30°, $y°$ = 60° d)$x°$ = 260°, $y°$ = 100°,
50° **10.a)**43° b)109° **11.a)**160° b)3.05 cm

xercise, page 333
R = 110°, ∠S = 95° b)∠P = 75°,
153° **4.a)**y = 55° b)x = 125° **5.a)**60°
° c)10° **6.a)**$x°$ = 30°, $y°$ = 70°, $z°$ = 30°
= 20°, $y°$ = 90°, $z°$ = 90° c)$x°$ = 50°,
70°, $z°$ = 70° d)$y°$ = 125°

xercise, page 341
$\sqrt{2}$ cm (ii)90° b)50° c)16 cm d)45° **2.a)**5
= $y°$ = 20° c)$x°$ = 140° **3.**16 cm **4.b)**8 cm
cm

xercise, page 344
° b)40° c)$x°$ = 65°, $y°$ = 42° d)$x°$ = 85°,
23°, $z°$ = 23° **2.a)**50° b)100° c)30° **3.a)**50°
c)140° **4.a)**50° b)60°

xercise, page 349
b)2 c)2 d)3.53 e)1.25 f)2.5 g)$\frac{1}{3}$ **2.**1 cm
m 19.5 cm **5.**24 cm **6.**CD = 3.6 cm,
6.0 cm **7.**PT = 9.8 cm, RQ = 4.0 cm

ercise, page 352
cm b)18.8 cm c)18.4 cm **2.a)**3.9 cm
m c)11.2 cm **3.a)**0.5 rad b)1.9 rad c)5.0 rad
7 cm² b)23.5 cm² c)1028.8 cm² d)163.3 cm²
cm² f)395.1 cm² g)148.8 cm² h)20.3 cm²
cm b)12.5 cm c)17.4 cm d)2.1 cm **6.a)**11.5 cm
m c)22.5 cm d)3.9 cm **7.a)**30.0° b)3.7 rad
° d)2.5 rad **8.**4 m **9.**0.125 rad **10.a)**0.4 rad
² **11.**175° **12.a)**37.3 cm² b)6.4 cm **13.**4.6 cm
4 cm² b)3.7 cm²

TER 9

rcise, page 359
b)3^{16} c)4^6 d)9^6 **3.a)**3^{15} b)3^{12} c)3^9 d)3^8 e)3^6
g)3^{mn} h)3^{3m} i)3^{2n} j)3^{3+m} **4.a)**32 b)6 c)−64

b)y^{a+b-c} c)a^2 d)x^{2b} e)a^{x+2y+7} f)$a^{x+y}b^{2x}$ **10.a)**B
b)A **11.a)**$\frac{a^5c}{b^6}$ b)1 c)$x^{4a}y^{4b}$ **12.a)**36 b)4 c)$\frac{1}{4}$ **14.b)**0, 1

9.2 Exercise, page 363
1.a)1 b)$\frac{1}{49}$ c)$\frac{10\,000}{9}$ d)$-\frac{1}{27}$ e)$-\frac{1}{27}$ f)−27 g)$\frac{1}{27}$
h)−1 i)−100 000 j)1 **2.a)**$\frac{2}{3}$ b)$\frac{5}{6}$ c)$\frac{12}{7}$ d)$\frac{1}{3}$ e)$\frac{1}{4096}$
f)$\frac{1}{729}$ g)−8 h)1 i)1 j)2 k)1 l)4 m)1 n)1 **3.a)**x^2 b)y^2
c)$\frac{1}{x^8}$ d)z^6 e)1 f)$\frac{32}{a^5b^5}$ g)$\frac{1}{a}$ **4.a)**$\frac{x^3}{y^2}$ b)$\frac{3py}{x}$ c)$\frac{3x}{4}$ d)$\frac{p^2s^2}{3b}$
e)$\frac{4^2s}{5p}$ f)$\frac{n}{x^3y^2}$ **5.a)**$\frac{q^2}{p^3}$ b)$\frac{3}{x^2}$ c)$\frac{n^4}{16m^4}$ d)$\frac{1}{9x^2}$ e)$\frac{a^4}{9}$ f)$2187w^7$
g)$7y^2$ h)$\frac{w^3}{5}$ **6.a)**1 b)$\frac{1}{v^6}$ c)$\frac{b^2}{243}$ d)k^8 e)$\frac{156}{d^{12}}$ f)$\frac{1}{2z^5}$ g)$\frac{xy^{12}}{3}$
h)$\frac{7}{m^5}$ i)$\frac{4}{9s^2}$ **7.a)**m^{2x+y} b)s^{3x-2p} c)t^{5x} d)q^{x-1} e)y^{2n-m}
f)k^{5m+3n} g)g^{2a+2b} h)q^{a-b-2c} **8.a)**$3 + \frac{3}{q^4}$ b)$\frac{(m+n)^2}{mn}$
c)$\frac{p-2}{p+2}$ **9.c)**−1024 b)−4 c)−1024 **10.a)**$\frac{7}{10}$ b)$\frac{3}{56}$
c)−32 d)$\frac{507}{64}$ e)$-\frac{2}{3}$ f)0 **11.a)**$\sqrt{7}$ b)$\sqrt[5]{x}$ c)$\sqrt[4]{a}$ d)$\sqrt[5]{8^2}$
e)$\sqrt[7]{b^3}$ f)$5\sqrt[3]{x^2}$ g)$\frac{1}{\sqrt[3]{x}}$ h)$\frac{1}{\sqrt[5]{a^2}}$ **12.a)**$6^{\frac{1}{2}}$ b)$10^{\frac{1}{2}}$ c)$7^{\frac{1}{3}}$ d)$12^{\frac{3}{2}}$
e)$8^{\frac{9}{7}}$ f)7^{-1} **13.a)**$x^{\frac{1}{3}}$ b)$x^{-\frac{1}{3}}$ c)$x^{-\frac{3}{2}}$ d)$a^{\frac{5}{6}}$ e)$a^{-\frac{1}{4}}$ f)$a^{\frac{2}{3}}$ **14.a)**$\sqrt[4]{3^3}$
b)$\sqrt[9]{5^4}$ c)$b\sqrt{a}$ d)a^2b^3 e)xy^2z^3 f)$y + 2\sqrt{y^3}$ g)$2x^3y^2$ h)$\sqrt{8}$
15.a)3 b)4 c)27 d)8 e)32 f)2 g)27 h)4 **16.a)**8 b)$\frac{1}{9}$
c)$\frac{1}{32}$ d)$\frac{1}{27}$ e)1000 f)0 g)$\frac{1}{5}$ h)$\frac{3125}{32}$ i)5 j)2 **17.**C $\frac{1}{32}$,
A $\frac{1}{9}$, D $\frac{1}{2}$, B 36

9.3 Exercise, page 368
(Last digit of answers may vary)
1.a)2.5 b)22.6 c)13.9 d)6.5 e)9.9 f)34.3 **2.a)**10.0
b)52.1 c)5.2 **3.a)**20.0 b)63.1 c)31.6 **4.a)**2.6 b)1.8
c)1.4 d)1.8 e)3.4 **5.a)**15.8 b)1.7 c)31.0 d)1.8 e)63
7.c)$(x, y) \rightarrow (-x, y)$ **8.c)**$(x, y)(-x, y)$ **16.b)**decay
17.a)2.5 b)4.7 c)14.5 **18.a)**95 b)53 c)13 **19.b)**growth
c)(i)$440 (ii)$495 (iii)$600 d)(i)2.5 years (ii)3.6 years

9.4 Exercise, page 372
Answers may vary for question 1 to 7.
1.a)10.1 b)10.1 c)Approximately the same. **2.a)**2.3
b)2.2 c)Approximately the same. **3.a)**22 b)2.6 c)24.1
d)17.5 e)8.0 **4.a)**22 b)2.7 c)24.5 d)17.7 e)8.1