Lesson Plan
Grade 9 Academic
Lesson: __45__
Unit/Chapter: Properties of 2-D Figures
Topic: Quadrilateral and Triangle Properties

- **homework check:** NPM 9 p. 394 #2, 3, 5, 7, 8, 10

- **note:** Quadrilateral and Triangle Properties

In the following investigations, you will make a conjecture about the relationship and use the given diagrams to mathematically prove or reject your hypothesis. A complete solution will include diagrams, calculations, and an organized process outlining a conclusion.

- **homework assignment:** Relationships between Areas of Polygons
Diagonals of Quadrilaterals

Square – a quadrilateral with equal sides and equal angles
Rectangle – a quadrilateral with two pairs of opposite sides equal and equal angles
Rhombus – a parallelogram with two pairs of adjacent sides equal
Parallelogram – a quadrilateral with opposite sides parallel
Isosceles Trapezoid – a quadrilateral with exactly one pair of parallel sides
Kite – a quadrilateral with two pairs of equal adjacent sides

- The diagonals of certain quadrilaterals have special properties:

<table>
<thead>
<tr>
<th>Type of Quadrilateral</th>
<th>The diagonals...</th>
<th>The diagonals form angles that are...</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>square</td>
<td>are equal and bisect each other.</td>
<td>all 90°.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>rhombus (not a square)</td>
<td>are not equal and bisect each other.</td>
<td>all 90°.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>rectangle (not a square)</td>
<td>are equal and bisect each other.</td>
<td>equal when opposite and supplementary when adjacent.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>parallelogram (not a rectangle or rhombus)</td>
<td>are not equal and bisect each other.</td>
<td>equal when opposite and supplementary when adjacent.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>isosceles trapezoid (not a rectangle or rhombus)</td>
<td>are equal and intersect to form two pairs of equal line segments.</td>
<td>equal when opposite and supplementary when adjacent.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>kite</td>
<td>may or may not be equal and only one is bisected by the other.</td>
<td>all 90°.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Need to Know

- You can identify the type of quadrilateral by using its diagonal properties.

Bimedian – the line joining the midpoint of opposite sides
Trapezoid Bimedian – the bimedian of the non-base side is parallel to the base

- length of bimedian is \( \frac{\text{top length} + \text{base length}}{2} \)

Length/Distance Formula: \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
**Midsegments of Quadrilaterals**

midsegment – line joining the midpoints of adjacent sides
inscribed – completely contained within another shape intersecting in a singular point with any side or curve
circumscribed – completely surrounding another shape intersecting every vertex exactly once

- The midsegments of any quadrilateral form a parallelogram:

<table>
<thead>
<tr>
<th>Midsegments Form a Parallelogram</th>
<th>Midsegments Form a Rhombus</th>
<th>Midsegments Form a Rectangle</th>
<th>Midsegments Form a Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>• parallelogram</td>
<td>• rectangle</td>
<td>• rhombus</td>
<td>• square</td>
</tr>
<tr>
<td>• trapezoid</td>
<td>• isosceles trapezoid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• irregular quadrilateral</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Triangle Centres**

Median – line from the vertex to the midpoint of the opposite side
Centroid – the centre of a circle located at the intersection of the medians
Angle bisector – line that cuts the measure of an angle in two equal parts
Incentre – the centre of a circle located at the intersection of the angle bisectors
Altitude – the height of a triangle drawn from the vertex perpendicular to the opposite side
Orthocentre – the centre of a circle located at the intersection of the altitudes
Perpendicular bisector – line from the vertex to the opposite side at right angles
Circumcentre – the centre of a circle located at the intersection of the perpendicular bisectors

**Midpoint Formula:** \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)
The Relationship between Area of Polygons

Triangles

How is the area of an inscribed triangle related to the circumscribed triangle?

Hypothesis (2):

Proof:

Use points \(A(-1, 5), B(-1, -1), \) and \(C(3, -1)\) and midsegments to prove your hypothesis.

Quadrilaterals

How is the area of an inscribed quadrilateral related to the circumscribed quadrilateral?

Hypothesis (2):

Proof:

Use points \(A(-3, 3), B(-1, 5), C(7, 5)\) and \(D(1, -1)\) and midsegments to prove your hypothesis.
Triangle Graph (6):

Quadrilateral Graph (8):
Calculations for Triangle Areas (10):
Calculations for Quadrilateral Areas (10):
*be sure to use pythagorean formula correctly!