

## LESSON PLAN

Course: Grade 12 U Advanced Functions

Lesson : 1 - 10

Unit/Chapter: Polynomial Skills

Topic: Solving Polynomial Equations

---

▣ **homework check:** FM12 p. 38 exercise 1.14 # 3 & 5 plus handout

▣ **note:** Solving Polynomial Equations

To solve a polynomial equation, we use a combination of the factor and remainder theorems as well as synthetic division to factor the polynomial fully. Once factored, we set each of our factors to zero or use the quadratic equation to find the roots.

examples)

$$\text{Solve: } 2x^3 - 3x^2 - 5x + 6 = 0$$

Use factor theorem to find an initial factor.

$$P(x) = 2x^3 - 3x^2 - 5x + 6$$

$$P(1) = 2(1)^3 - 3(1)^2 - 5(1) + 6 \quad \text{Therefore, } (x-1) \text{ is a factor.}$$

$$P(1) = 0$$

Use synthetic division to find other factors.

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -5 & 6 \\ & & 2 & -1 & -6 \\ \hline & 2 & -1 & -6 & 0R \end{array}$$

$$\begin{aligned} \text{Therefore, } 2x^3 - 3x^2 - 5x + 6 &= (x-1)(2x^2 - x - 6) \\ &= (x-1)(2x+3)(x-2) \end{aligned}$$

$$(x-1)(2x+3)(x-2) = 0 \text{ and therefore,}$$

$$\begin{array}{lll} x-1=0 & 2x+3=0 & x-2=0 \\ x=1 & x=\frac{-3}{2} & x=2 \end{array}$$

Solve:  $x^4 - 5x^2 + 4 = 0$

Use the remainder and factor theorems to find a single factor:

$$P(x) = x^4 - 5x^2 + 4$$

$$P(1) = (1)^4 - 5(1) + 4$$

$$P(1) = 0$$

Use synthetic division to get:

$$(x-1)(x^3 + x^2 - 4x - 4) = 0$$

Set a new  $P(x)$  to factor the new cubic:

$$P(x) = x^3 + x^2 - 4x - 4$$

$$P(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$$

$$P(-1) = 0$$

Use synthetic division to get:

$$(x-1)(x+1)(x^2 - 4) = 0$$

$$(x-1)(x+1)(x-2)(x+2) = 0$$

By setting each factor to zero, we get four solutions of:

$$x-1=0 \quad x+1=0 \quad x-2=0 \quad x+2=0$$

$$x=1 \quad x=-1 \quad x=2 \quad x=-2$$

▣ **note:** FM12 p. 43 exercise 1.16 # 1, 3, 4

## EXERCISE 1.16

1. Solve.

(a)  $x^3 - 3x^2 - 4x + 12 = 0$

(b)  $x^3 - 4x^2 + x + 6 = 0$

(c)  $t^3 - t^2 - 16t - 20 = 0$

(d)  $3w^3 + w^2 - 3w - 1 = 0$

(e)  $2z^3 - z^2 - 15z + 18 = 0$

(f)  $4t^3 - 7t^2 - 21t + 18 = 0$

(g)  $2x^3 - 9x^2 - 8x + 15 = 0$

(h)  $x^3 - 3x^2 - 16x + 48 = 0$

3. Solve.

(a)  $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

(b)  $x^4 - x^3 - 13x^2 + x + 12 = 0$

(c)  $x^4 - 1 = 0$

(d)  $6t^3 + 7t^2 - 43t - 30 = 0$

4. Solve.

(a)  $x^4 - 27x = 0$

(b)  $x^2(x + 1) = 12 + 8x$

- (i)  $m = 0$  or  $m = 4$       (j)  $x = \frac{5}{3}$  or  $x = -\frac{3}{2}$
3. (a)  $\frac{-7 \pm \sqrt{13}}{6}$       (b)  $-3 \pm \sqrt{5}$       (c)  $\frac{3 \pm \sqrt{2}}{2}$       (d)  $\frac{3 \pm \sqrt{7}}{2}$
- (e)  $m = 0$  or  $m = \frac{7}{3}$       (f)  $\frac{\pm\sqrt{22}}{2}$       (g)  $\frac{-3 \pm \sqrt{15}i}{4}$       (h)  $m = -\frac{4}{3}$  or  $m = \frac{3}{2}$
- (i)  $\frac{-3 \pm \sqrt{5}}{2}$       (j)  $\frac{3 \pm \sqrt{21}}{3}$
4. (a)  $x = 1.16$  or  $-5.16$       (b)  $t = -0.38$  or  $-2.62$       (c)  $m = 3.24$  or  $-1.24$   
 (d)  $x = -0.68$  or  $-7.32$       (e)  $x = 0.29$  or  $-2.29$       (f)  $m = 0.61$  or  $-0.47$   
 (g)  $x = 0.29$  or  $-0.69$       (h)  $y = 5.85$  or  $-0.85$       (i)  $m = 0.28$  or  $-1.78$   
 (j)  $x = 0.62$  or  $-1.62$
5. (a)  $x = -\frac{1}{2}$  or  $\frac{1}{3}$       (b)  $\frac{-1 \pm \sqrt{73}}{6}$       (c)  $\frac{1 \pm \sqrt{7}}{2}$       (d)  $\frac{-7 \pm \sqrt{73}}{4}$       (e)  $\frac{-11 \pm \sqrt{61}}{2}$
- (f)  $\frac{3 \pm \sqrt{5}}{2}$       (g)  $\frac{14 \pm 4\sqrt{71}}{47}$       (h)  $\frac{\pm\sqrt{21}}{7}$       (i)  $\frac{75 \pm \sqrt{4345}}{20}$
6. (a)  $\frac{1 \pm \sqrt{17}}{14}$       (b)  $1 \pm \sqrt{5}$       (c)  $-2 \pm \sqrt{2}$       (d)  $0$  or  $6$       (e)  $\frac{3 \pm \sqrt{129}}{4}$       (f)  $\pm 9$
7. (a)  $x = \frac{-m \pm \sqrt{m^2 + 4t}}{2}$       (b)  $x = \pm \sqrt{q}$       (c)  $x = \frac{-e}{d}$       (d)  $x = \frac{\pm\sqrt{rs}}{r}$
8. (a)  $6, -\frac{2}{7}$       (b)  $\frac{3}{2}, 5$       (c)  $\frac{2 \pm \sqrt{58}}{3}$       (d)  $0.1, -0.15$       (e)  $\sqrt{6}, -\frac{\sqrt{6}}{3}$

### EXERCISE 1.15

1. (a)  $7 + 3i$       (b)  $4$       (c)  $7$       (d)  $5 - 6i$       (e)  $21 - 4i$
2. (a)  $3i$       (b)  $12i$       (c)  $2i, 5$       (d)  $5i, 3$       (e)  $-7$
- (f)  $2i, 6$       (g)  $7$       (h)  $10i, 2$       (i)  $0$       (j)  $1$
3. (a)  $5$       (b)  $2$       (c)  $11 + 7i$       (d)  $16 + 11i$       (e)  $19 + 9i$
- (f)  $22 + 4i$       (g)  $29$
4. (a)  $\frac{3}{2} \pm \frac{\sqrt{7}}{2}i$       (b)  $-\frac{1}{6} \pm \frac{\sqrt{47}}{6}i$

### EXERCISE 1.16

1. (a)  $x = -2$  or  $2$  or  $3$       (b)  $x = -1$  or  $2$  or  $3$       (c)  $t = -2$  or  $t = 5$   
 (d)  $w = -1$  or  $-\frac{1}{3}$  or  $1$       (e)  $z = -3$  or  $\frac{3}{2}$  or  $2$       (f)  $t = -2$  or  $\frac{3}{4}$  or  $3$   
 (g)  $x = -\frac{3}{2}$  or  $1$  or  $5$       (h)  $x = -4$  or  $3$  or  $4$
2. (a)  $x = 2$ , or  $-1 \pm \sqrt{2}$       (b)  $t = -1$  or  $\frac{3 \pm \sqrt{5}}{2}$       (c)  $w = -2$  or  $1 \pm \sqrt{2}i$
- (d)  $x = 1$  or  $\frac{-1 \pm \sqrt{3}i}{2}$       (e)  $t = -2$  or  $1 \pm \sqrt{3}i$       (f)  $x = -3$  or  $\frac{1 \pm \sqrt{21}}{2}$
3. (a)  $x = -1$  or  $2$  or  $\pm 3$       (b)  $x = -3$  or  $4$  or  $\pm 1$   
 (c)  $x = \pm 1$  or  $\pm i$       (d)  $t = -3$  or  $-\frac{2}{3}$  or  $\frac{5}{2}$
4. (a)  $x = 0$  or  $3$  or  $\frac{-3 \pm 3\sqrt{3}i}{2}$       (b)  $x = -2$  or  $x = 3$
5. (a)  $x^2 - 12x + 35 = 0$       (b)  $x^3 - 6x^2 + 5x + 12 = 0$   
 (c)  $x^3 - 9x^2 + 8x + 60 = 0$       (d)  $x^3 + 9x^2 + 23x + 15 = 0$   
 (e)  $x^3 - 4x^2 + 3x + 2 = 0$
6. (a)  $x = -3$  or  $\pm 2$       (b)  $x = -3$  or  $\pm 1$  or  $\pm 2$       (c)  $\frac{1}{3}, \pm\sqrt{2}$   
 (d)  $\frac{1}{2}, 1 \pm \sqrt{2}$       (e)  $-1, 2, -1 \pm i\sqrt{2}$       (f)  $-\frac{1}{3}, 3, \frac{-1 \pm i\sqrt{7}}{2}$
7. (a)  $x^2 - 2x - 1 = 0$       (b)  $x^2 - 4x - 3 = 0$       (c)  $x^2 - 2x + 10 = 0$       (d)  $x^2 - 4x + 40 = 0$