LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson : __1 - 6__**

Unit/Chapter: __Polynomial Skills__ **Topic: _Remainder and Factor Theorems_**

- ***homework check:*** **FM12** p. 25 exercise 1.8

- ***note:*** **Remainder and Factor Theorems**

The remainder and factor theorems work together to help us find factors of larger polynomials. Recall that a factor must divide evenly (without remainder – or remainder equal to zero) into the given number, or in our case, the given polynomial. If the polynomial does not divide evenly, there will be a remainder, and therefore, that polynomial is not a factor.

To use the remainder and factor theorems, we use the root as the "x" value and substitute into the polynomial. If the root is a factor, the remainder will be zero. For example,

to determine whether $(x-3)$ is a factor of $P(x)=x^3-x^2-7x+9$, we use the root of positive 3 and substitute $P(3)$

$P(3)=(3)^3-(3)^2-7(3)+9$

$P(3)=27-9-21+9$

$P(3)=6$

Since the remainder $\neq 0$, $(x-3)$ is not a factor of $P(x)=x^3-x^2-7x+9$ and if we used long or synthetic division, we would get a remainder of 6.

To factor any polynomial of degree larger than 2, we must first find a factor. This can sometimes be a long process of guess and check. We use the remainder/factor theorems to quicken the process. This establishes the first factor and allows us to find the remaining factors using synthetic division. For example, Factor $4x^3+16x^2+9x-9$

$$P(1)=4(1)^3+16(1)^2+9(1)-9$$
$$P(1)=20$$
$$P(-1)=4(-1)^3+16(-1)^2+9(-1)-9$$
$$P(-1)=-6$$
$$P(3)=4(3)^3+16(3)^2+9(3)-9$$
$$P(3)=270$$
$$P(-3)=4(-3)^3+16(-3)^2+9(-3)-9$$
$$P(-3)=0$$

Therefore, $x=-3$ which means that $(x+3)$ is a factor of P(x)

To find the remaining factors, we must first divide to get the remaining polynomial.

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 9 & -9 \\ & & -12 & -12 & 9 \\ \hline & 4 & 4 & -3 & 0R \end{array}$$

$$\begin{aligned} 4x^3+16x^2+9x-9 &= (x+3)(4x^2+4x-3) \\ &= (x+3)(2x+3)(2x-1) \end{aligned}$$

- ***homework assignment:*** **FM12** p. 29 exercise 1.10 #1, 6, 8 -10, 13

EXERCISE 1.10

1. Determine which of the polynomials have $x - 1$ as a factor.
(a) $x^3 + x^2 - x - 1$
(b) $2x^3 - x^2 - 3x - 1$
(c) $x^4 - 3x^3 + 2x^2 - x + 1$
(d) $3x^3 - x - 3$
(e) $4x^4 - 2x^3 + 3x^2 - 2x + 1$
(f) $x^3 - 3x^2 + 4x - 2$
(g) $2x^3 + 4x^2 - 5x - 1$
(h) $x^3 - x^2 - x - 1$

6. Factor the following over the integers.
(a) $x^3 - 6x^2 + 11x - 6$
(b) $x^3 + 8x^2 + 19x + 12$
(c) $t^3 - 2t^2 - 9t + 18$
(d) $m^3 + 4m^2 + 2m - 3$
(e) $x^3 + x^2 - 22x - 40$
(f) $x^3 + x^2 - 16x - 16$
(g) $w^3 - 2w^2 - 6w - 8$
(h) $n^3 + 6n^2 - 7n - 60$
(i) $x^3 - 27$
(j) $x^3 - 27x + 10$

8. Find k so that $x^3 - 2x^2 + 3x + k$ has $(x - 1)$ as a factor.

9. Find k so that $x^3 + 5x^2 + kx + 6$ has $(x + 2)$ as a factor.

10. Find k so that $km^3 - 10m^2 + 2m + 3$ has $(m - 3)$ as a factor.

13. Find the value of k so that when $x^2 + 8x + k$ is divided by $x - 2$ the remainder is 3.