

LESSON PLAN

Course: Grade 12 U Advanced Functions

Lesson : 1 - 7

Unit/Chapter: Polynomial Skills

Topic: Sums and Differences of Cubes

✚ **homework check:** FM12 p. 29 exercise 1.10 #1, 6, 8 -10, 13

✚ **notes:** Sums and Differences of Cubes

A sum of cubes has the form $a^3 + b^3$ while a differences of cubes has the form $a^3 - b^3$. In order to factor a sum or difference of cubes, we can rely on the factor theorem to establish the first factor and then use synthetic division, or we can use the established patterns that go with either a sum or difference of cubes.

In general, once we have recognized the sum or difference of cubes, we can use

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

example) Factor.

$$\begin{aligned} 125x^3 - 64y^3 &= \\ &= (5x)^3 - (4y)^3 \\ &= (5x - 4y)(25x^2 + 20xy + 16y^2) \end{aligned}$$

$$\begin{aligned} 1000a^3 + 27b^3 &= \\ &= (10a)^3 + (3b)^3 \\ &= (10a + 3b)(100a^2 - 30ab + 9b^2) \end{aligned}$$

✚ **homework assignment:** FM12 p. 31 exercise 1.11 #2 and p. 29 exercise 1.10 #7 (from yesterday's handout)

EXERCISE 1.11

2. Factor.

(a) $x^3 - 27$

(c) $m^3 - 8$

(e) $8x^3 - 1$

(g) $27x^3 + 8y^3$

(i) $s^6 - 27t^9$

(k) $a^3b^3 + 27c^3$

(m) $\frac{1}{8}x^3 - 1$

(b) $x^3 + 64$

(d) $y^3 + 125$

(f) $64m^3 - 1$

(h) $x^6 + y^9$

(j) $1000x^{12} + y^{15}$

(l) $x^3y^6 - 9z^9$

(n) $m^6 - \frac{1}{27}n^3$

EXERCISE 1.11

2. (a) $(x - 3)(x^2 + 3x + 9)$
 (c) $(m - 2)(m^2 + 2m + 4)$
 (e) $(2x - 1)(4x^2 + 2x + 1)$
 (g) $(3x + 2y)(9x^2 - 6xy + 4y^2)$
 (i) $(s^2 - 3t^3)(s^4 + 3t^3 + 9t^6)$
 (k) $(ab + 3c)(a^2b^2 - 3abc + 9c^2)$
 (m) $\left(\frac{x}{2} - 1\right)\left(\frac{x^2}{4} + \frac{x}{2} + 1\right)$

4. (a) no (b) yes
 5. yes

- (b) $(x + 4)(x^2 - 4x + 16)$
 (d) $(y + 5)(y^2 - 5y + 25)$
 (f) $(4m - 1)(16m^2 + 4m + 1)$
 (h) $(x^2 + y^3)(x^4 - x^2y^3 + y^6)$
 (j) $(10x^4 + y^5)(100x^8 - 10x^4y^5 + y^{10})$
 (l) $(xy^2 - \sqrt[3]{9z^3})(x^2y^4 + \sqrt[3]{9xy^2z^3} + \sqrt[3]{81z^6})$
 (n) $\left(m^2 - \frac{n}{3}\right)\left(m^4 + \frac{m^2n}{3} + \frac{n^2}{9}\right)$

EXERCISE 1.12

1. (a) 16 (b) 13 (c) 38
 2. (a) $2]x - 3]x + 5; 7$
 (c) $4]x - 2]x + 4]x + 1; 33$
 (e) $2]x - 2]x - 3]x + 11; 13$
 3. (a) $-2]x + 4]x + 3; -3$
 (c) $-1]x - 2]x + 2]x + 9; -30$
 (e) $-1]x - 1]x - 1]x - 1; -40$
 4. (a) $2]x - 3]x - 4]x + 5]x + 2; 278$
 (c) $-2]x - 3]x + 4]x - 1]x - 3; -647$
 5. (a) $3]x - 4]x - 5]x - 1; -31$
 (c) $-1]x - 2]x + 3]x + 9; 3$
 (e) $-5]x - 2]x + 4]x - 3]x + 18; -24$

- (d) 25
 (b) $3]x + 7]x - 5]x - 8; 34$
 (d) $1]x - 1]x - 5]x + 6; 0$
 (b) $-3]x + 1]x - 5]x + 10; -77$
 (d) $-4]x + 3]x - 1]x + 4; -80$
 (b) $3]x + 2]x + 1]x - 3]x + 8; 908$
 (d) $-1]x - 2]x + 3]x - 1]x + 12; -328$
 (b) $-2]x + 5]x - 1]x + 7; 45$
 (d) $2]x + 1]x - 3]x - 4]x - 11; 9$

EXERCISE 1.13

1. (a) $x = 3$ or $x = 4$ (b) $m = -3$ or $m = -2$ (c) $t = -9$ or $t = 2$
 (d) $y = 1$ or $y = -1$ (e) $m = \frac{1}{2}$ or $m = -5$ (f) $x = -\frac{7}{4}$ or $x = \frac{1}{3}$
 2. (a) -1 and -2 (b) -2 and 1 (c) -5 and 7 (d) -4 and -6
 3. (a) $(x + 3)(x - 2) = 0; x = -3$ or $x = 2$ (b) $(y + 3)(y + 4) = 0; y = -3$ or $y = -4$
 (c) $(t + 2)(t - 8) = 0; t = -2$ or $t = 8$ (d) $(w + 2)(w - 2) = 0; w = -2$ or $w = 2$
 (e) $(x + 5)(x + 8) = 0; x = -5$ or $x = -8$ (f) $(m + 2)(m - 7) = 0; m = -2$ or $m = 7$
 (g) $(x + 9)(x - 3) = 0; x = -9$ or $x = 3$ (h) $(m + 5)(m - 5) = 0; m = -5$ or $m = 5$
 (i) $(x - 4)^2 = 0; x = 4$ (j) $(t + 5)^2 = 0; t = -5$
 4. (a) $x = 3$ or $x = -\frac{5}{3}$ (b) $x = -\frac{1}{2}$ or $x = -\frac{2}{3}$ (c) $m = -\frac{4}{3}$ or $m = \frac{4}{3}$ (d) $y = \frac{1}{4}$ or $y = -$
 (e) $w = -5$ or $w = \frac{1}{2}$ (f) $x = -6$ or $x = 2$ (g) $y = -3$
 5. (a) $t = 3$ or $t = -\frac{1}{4}$ (b) $x = 0$ or $x = \frac{5}{3}$ (c) $m = \frac{2}{3}$ or $m = -5$ (d) $t = \frac{3}{2}$
 (e) $y = \frac{5}{3}$ or $y = -3$ (f) $m = 0$ or $m = \frac{5}{2}$ (g) $n = -\frac{1}{6}$ or $n = \frac{1}{3}$
 6. (a) $t = 2$ or $t = 5$ (b) $x = \frac{3}{5}$ or $x = -7$ (c) $t = -4$ or $t = 4$ (d) $x = 2$
 (e) no solutions (f) $x = 1$ or $x = 7$ (g) $x = -3, x = 6$

EXERCISE 1.14

1. (a) $a = 1, b = 3, c = 7$ (b) $a = 6, b = 8, c = -1$
 (c) $a = 4, b = -3, c = -9$ (d) $a = 2, b = -1, c = 11$
 (e) $a = 5, b = -1, c = -12$ (f) $a = 1, b = 1, c = 0$
 (g) $a = 1, b = 0, c = -16$ (h) $a = 2, b = -4, c = -9$
 2. (a) $y = 6$ or $y = -5$ (b) $t = -\frac{4}{5}$ or $t = 1$ (c) $m = -\frac{5}{3}$ or $m = 4$ (d) $x = -\frac{1}{4}$ or $x =$
 (e) $m = 1$ or $m = \frac{5}{4}$ (f) $x = \frac{7}{3}$ or $x = -4$ (g) $t = -\frac{5}{2}$ or $t = 3$ (h) $r = \frac{5}{3}$ or $r = -$