LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: 2 - 5**

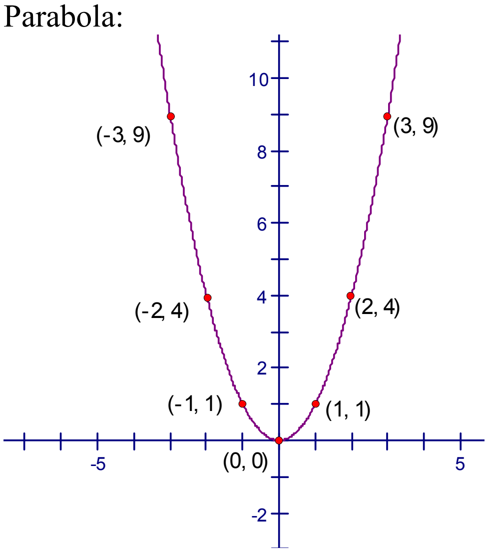
Unit/Chapter: Functions **Topic: Inverse Functions**

- ***homework check:*** **FM12** exercise 6.4

- ***note:*** **Inverse Functions**

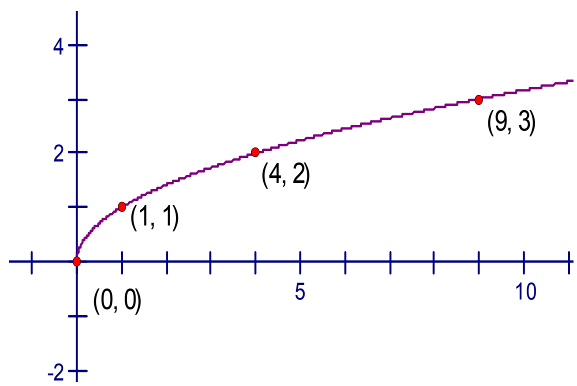
A function has an inverse if $f(x)$ maps x onto y and $f^{-1}(x)$ maps y back onto x, in other words $f^{-1}(x)$ undoes what $f(x)$ originally accomplished. A function has an inverse if and only if it is a 1 – 1 function. To determine if a function is 1 – 1, we can use the horizontal line test. For example,

Parabola:



*not a 1 – 1 function

Root:



*1 – 1 function

To find the inverse algebraically, we rely on the idea that if $f(x)$ maps x onto y and $f^{-1}(x)$ maps y back onto x, then the inverse is exchanging the x variable with the y variable. For example,

$y = 4x - 3$ To find the inverse, we exchange x and y: $x = 4y - 3$

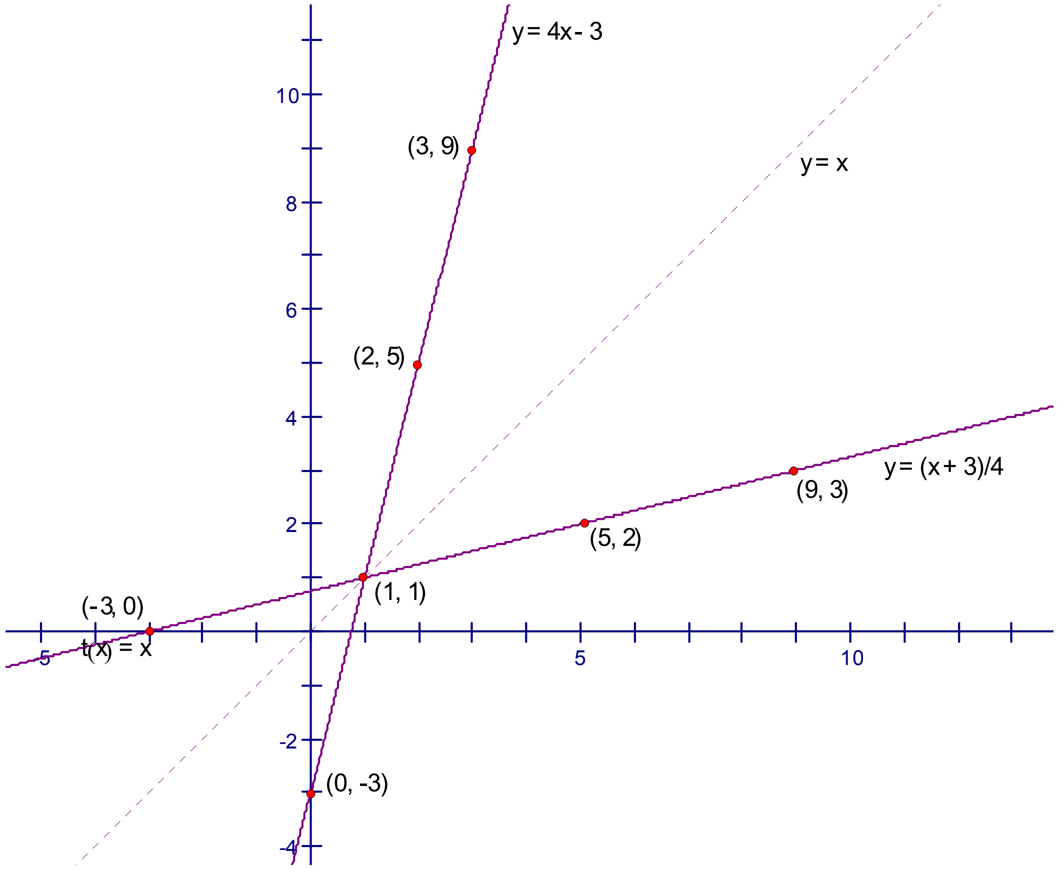
$x + 3 = 4y$

and solve for y: $\frac{x+3}{4} = y$

$\therefore f^{-1}(x) = \frac{x+3}{4}$

The principle of interchanging x and y also gives us a way to graph the inverse, given the original. For example,

Graph $y = \frac{1}{2}x + 3$ and its inverse.

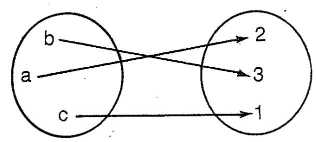


⌑ ***homework assignment:*** **<u>FM12</u>** p. 207 exercise 6.8 #2, 3, 5, 6, & 8

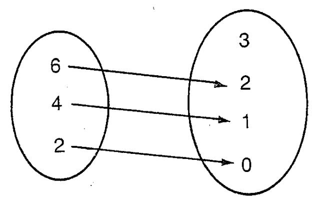
EXERCISE 6.8

A 1. Which of the functions represented by the following arrow diagrams are 1-1?

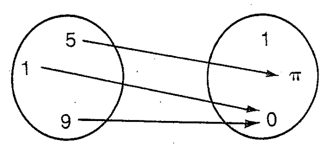
(a)



(b)

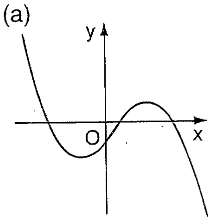


(c)

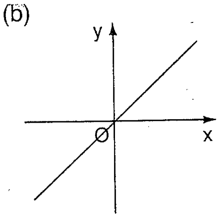


2. Which of the functions whose graphs are given are one-to-one functions?

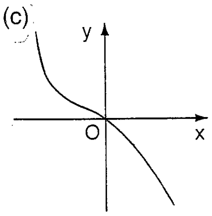
(a)



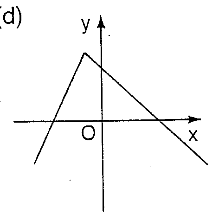
(b)



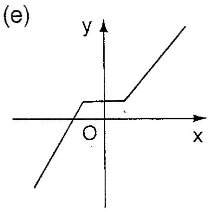
(c)



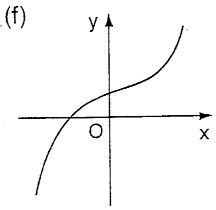
(d)



(e)



(f)



B 3. Which of the following functions are 1-1?

(a) $f(x) = x + 1$ (b) $g(x) = |x|$

(c) $y = 3 - 2x$ (d) $h(x) = \frac{1}{x}$

(e) $F(x) = \frac{1}{x^2}$ (f) $y = 1 - x^2$

(g) $f(t) = -t^3$ (h) $G(t) = t^4$

(i) $y = \sqrt{x}$ (j) $f(x) = \frac{1}{x^2}, x < 0$

4. Draw arrow diagrams for the inverses of those functions in question 1 that are 1-1.

5. In each of the following cases find f^{-1} and state the domain and range of f^{-1}.

(a) $f(x) = 2 - 5x$ (b) $f(x) = 13x + 6$

(c) $f(x) = x^2, x \geqslant 0$ (d) $f(x) = \frac{1}{x}$

(e) $f(x) = x^3$

(f) $f(x) = 3x - 2, 0 \leqslant x \leqslant 4$

6. Find the inverses of the following functions.

(a) $y = \frac{1}{2}(x - 7)$ (b) $y = \frac{1}{5}(36 - x)$

(c) $y = 5x^3 - 6$ (d) $y = \sqrt{x}$

(e) $y = \sqrt{x - 3}$ (f) $y = 1 + \frac{1}{x}$

(g) $y = \frac{1}{1 + x}$ (h) $y = \frac{1 - x}{1 + x}$

(i) $y = \frac{4x - 1}{3x + 2}$ (j) $y = \frac{\pi - 3x}{x}$

(k) $y = x^4, x \geqslant 0$

(l) $y = 3(x - 1)^2, x \geqslant 1$

(m) $y = \sqrt{x^2 + 9}, x \geqslant 0$

(n) $y = \sqrt{25 - x^2}, x \geqslant 0$

7. In each of the following cases find f^{-1} and then calculate $f \circ f^{-1}$ and $f^{-1} \circ f$.

(a) $f(x) = 5x - 8$ (b) $f(x) = \sqrt{x}$

8. For each of the following functions,

(a) draw the graph of f.

(b) use it to draw the graph of f^{-1}.

(c) find the expression for $f^{-1}(x)$.

(i) $f(x) = 2x + 1$ (ii) $f(x) = x^2 + 2, x \geqslant$ (

(iii) $f(x) = x^3$ (iv) $f(x) = -\frac{1}{x}$

$(f \circ g)(x) = 3 - x^2$ on domain of g: $\{x \mid -1 \leqslant x \leqslant 1\}$

8. $f(x) = \sqrt{x}$, $g(x) = 8x^2 + x$
9. $g(x) = 4x - 5$
10. $g(x) = x^2 + x - 1$
11. $g(x) = 4x - 17$
12. (b) $(f \circ f)(x) = x + \dfrac{1}{x} + \dfrac{x}{x^2 + 1}$, $(f \circ f \circ f)(x) = x + \dfrac{1}{x} + \dfrac{x}{x^2 + 1} + \dfrac{x^3 + x}{x^4 + 3x^2 + 1}$
13. (a) $(f \circ g)(x) = \sin(5x)$, $(g \circ f)(x) = 5 \sin x$, $(f \circ f)(x) = \sin(\sin x)$, $(g \circ g)(x) = 25x$
    (b) $(f \circ g)(x) = \cos^2 x + 3$, $(g \circ f)(x) = \cos(x^2 + 3)$, $(f \circ f)(x) = x^4 + 6x^2 + 12$, $(g \circ g)(x) = \cos(\cos x)$
14. (a) domain of f: $\{x \mid x \leqslant -\sqrt{2}$ or $x \geqslant \sqrt{2}\}$, range of f: $\{y \mid y \geqslant 0\}$
    domain of g: R, range of g: $\{y \mid -1 \leqslant y \leqslant 1\}$
    (b) $f \circ g$ not defined (range of g not contained in domain of f)
    $(g \circ f)(x) = \sin(\sqrt{x^2 - 2})$ on domain of f: $\{x \mid x \leqslant -\sqrt{2}$ or $x \geqslant \sqrt{2}\}$

EXERCISE 6.8

1. (a) and (b) only
2. (b), (c), and (f) only
3. (a), (c), (d), (g), (i), and (j) only
5. (a) $f^{-1}(x) = \dfrac{2 - x}{5}$ domain: R, range: R
   (b) $f^{-1}(x) = \dfrac{x - 6}{13}$ domain: R, range: R
   (c) $f^{-1}(x) = \sqrt{x}$ domain: $\{x \mid x \geqslant 0\}$, range: $\{y \mid y \geqslant 0\}$
   (d) $f^{-1}(x) = \dfrac{1}{x}$ domain: $\{x \mid x \neq 0\}$, range: $\{y \mid y \neq 0\}$
   (e) $f^{-1}(x) = x^{\frac{1}{3}}$ domain: R, range: R
   (f) $f^{-1}(x) = \dfrac{x + 2}{3}$ domain: $\{x \mid -2 \leqslant x \leqslant 10\}$, range: $\{y \mid 0 \leqslant y \leqslant 4\}$
6. (a) $y = 2x + 7$ (b) $y = -5x + 36$ (c) $y = \left(\dfrac{x + 6}{5}\right)^{\frac{1}{3}}$ (d) $y = x^2, x \geqslant 0$
   (e) $y = x^2 + 3, x \geqslant 0$ (f) $y = \dfrac{1}{x - 1}$ (g) $y = \dfrac{1}{x} - 1$ (h) $y = \dfrac{1 - x}{1 + x}$
   (i) $y = \dfrac{2x + 1}{4 - 3x}$ (j) $y = \dfrac{\pi}{x + 3}$ (k) $y = \sqrt[4]{x}, x \geqslant 0$ (l) $y = 1 + \sqrt{\dfrac{x}{3}}, x \geqslant 0$
   (m) $y = \sqrt{x^2 - 9}, x \geqslant 3$ (n) $y = \sqrt{25 - x^2}, 0 \leqslant x \leqslant 5$
7. (a) $f^{-1}(x) = \dfrac{x + 8}{5}$; $(f \circ f^{-1})(x) = x$, $(f^{-1} \circ f)(x) = x$
   (b) $f^{-1}(x) = x^2, x \geqslant 0$; $(f \circ f^{-1})(x) = x$, $(f^{-1} \circ f)(x) = x$
8. (i) (c) $f^{-1}(x) = \dfrac{x - 1}{2}$
   (ii) (c) $f^{-1}(x) = \sqrt{x - 2}$ $(x \geqslant 2)$
   (iii) (c) $f^{-1}(x) = \sqrt[3]{x}$
   (iv) (c) $f^{-1}(x) = -\dfrac{1}{x}$

EXERCISE 6.9

1. 9
2. $f_n(x) = x^{2^{n+1}}$
3. $f_{47}(2) = \frac{46}{47}$
4. 800
5. 119
6. 24
7. 3

6.10 REVIEW EXERCISE

1. (a) 7 (b) 0 (c) 14 (d) 10 (e) −1 (f) 98
   (g) −10 (h) −6 (i) −12 (j) −2 (k) 62 (l) $\pi^2 - 2$
2. (a) domain $\{-1, 0, 1\}$, range $\{5, 6, 7\}$, 1 − 1
   (b) domain $\{1, 2, 3, 4\}$, range $\{\pi, 2\pi, 3\pi\}$, not 1 − 1
   (c) domain $\{1, 2, 3\}$, range $\{5, 7, 9\}$, 1 − 1
   (d) domain $\{2, 4, 6\}$, range $\{1, 3\}$, not 1 − 1
3. (a), (c), and (d) are graphs of functions and (c) is 1 − 1.
4. (a) 5 (b) 3 (c) 7 (d) 7
   (e) 2 (f) −2 (g) 1 (h) 10
5. (a) translate f downward 4 units