

LESSON PLAN

Course: Grade 12 U Advanced Functions

Lesson: 3 - 2

Unit/Chapter: Exponents & Logarithms Topic: Exponential Equations

- homework check:** FM11 exercise 1.8 and 1.9 p. 17 – 19

- note:** Exponential Equations

Recall, in order to solve an exponential equation, we must have the bases equal. To achieve equal bases, we require a good understanding of our basic exponent rules. Once the bases are equal, we can then set the exponents equal and solve for x using regular equation solving processes.

examples)

$$4^{2x-1} = \frac{1}{16^{3x+2}}$$

$$4^{2x-1} = (16^{3x+2})^{-1}$$

$$4^{2x-1} = \left[(4^2)^{3x+2} \right]^{-1}$$

$$4^{2x-1} = 4^{-6x-4}$$

$$\therefore 2x-1 = -6x-4$$

$$8x = -3$$

$$x = \frac{-3}{8}$$

$$4 \cdot 3^{2x+3} = 108$$

$$3^{2x+3} = 27$$

$$3^{2x+3} = 3^3$$

$$\therefore 2x+3 = 3$$

$$2x = 0$$

$$x = 0$$

$$2^{x+3} - 2^x = 224$$

$$2^x (2^3 - 1) = 224$$

$$2^x (7) = 224$$

$$2^x = 32$$

$$2^x = 2^5$$

$$\therefore x = 5$$

$$3^{2x} - 6(3^x) - 27 = 0$$

$$\text{let } a = 3^x$$

$$a^2 - 6a - 27 = 0$$

$$(a-9)(a+3) = 0$$

$$a = 9 \text{ and } a = -3$$

$$\text{but } a = 3^x$$

$$3^x = 9 \text{ and } 3^x = -3$$

$$x = 2 \quad \text{extraneous}$$

- homework assignment:** FM11 exercise 1.12 p. 25 #3, 4 and

AW11 exercise 1.6 p. 51 #12

Equations such as $2^{2x-1} = 2^9$ are solved by setting the exponents equal to each other and solving the equation.

$$\begin{aligned} 2x - 1 &= 9 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

The bases must be the same.

This method of solving an exponential equation is based on the property that if $a^x = a^y$, then $x = y$, for $x \neq -1, 0, 1$.

EXAMPLE 1. Solve. $3^x = 81$

SOLUTION:

$$\begin{aligned} 3^x &= 81 \\ 3^x &= 3^4 \\ x &= 4 \end{aligned}$$

\therefore the solution is $x = 4$.

Check these solutions.

EXAMPLE 3. Solve. $5^{x-2} = 625$

SOLUTION:

$$\begin{aligned} 5^{x-2} &= 625 \\ 5^{x-2} &= 5^4 \\ x - 2 &= 4 \\ x &= 6 \end{aligned}$$

\therefore the solution is 6.

EXAMPLE 2. Solve. $4^{8x} = \frac{1}{16}$

SOLUTION:

$$\begin{aligned} 4^{8x} &= \frac{1}{16} \\ (2^2)^{8x} &= 2^{-4} \\ 2^{16x} &= 2^{-4} \\ 16x &= -4 \\ x &= \frac{-4}{16} \\ x &= -\frac{1}{4} \end{aligned}$$

\therefore the solution is $x = -\frac{1}{4}$.

EXERCISE 1.12

B 1. Solve the following equations for $x \in N$.

- | | |
|--------------------|-------------------|
| (a) $2^x = 32$ | (b) $3^x = 27$ |
| (c) $2^x = 64$ | (d) $5^x = 25$ |
| (e) $3^x = 81$ | (f) $7^x = 49$ |
| (g) $(-3)^x = -27$ | (h) $(-2)^x = -8$ |
| (i) $(-2)^x = 16$ | (j) $(-5)^x = 25$ |

2. Solve the following equations for $x \in N$.

- | | |
|--------------------------|------------------------|
| (a) $4^x = 256$ | (b) $6^{x+3} = 6^{2x}$ |
| (c) $9^x = 729$ | (d) $2^x = 16^4$ |
| (e) $2^x = 4^{x-1}$ | (f) $2(5^x) = 1250$ |
| (g) $9^{2x-6} = 3^{x+6}$ | (h) $4^{2x-1} = 64$ |
| (i) $1^x = 1$ | (j) $(-1)^x = 1$ |

3. Solve the following equations for $x \in I$.

- | | |
|--------------------|--------------------|
| (a) $6^{3x-6} = 1$ | (b) $2^{-x} = 128$ |
|--------------------|--------------------|

$$(c) 5^{4-x} = \frac{1}{5} \quad (d) (-1)^x = 1$$

$$\begin{aligned} (e) 3^{2-x} &= 1 & (f) 4^{3x} &= 64 \\ (g) 4^{x-1} &= 1 & (h) (-1)^{2x} &= 1 \\ (i) 7^{x-2} &= 49 & (j) 2^{-2x} &= 32 \end{aligned}$$

4. Solve the following for $x \in R$.

- | | |
|-----------------------|----------------------------------|
| (a) $4^x = 8$ | (b) $2^{9x} = \frac{1}{8}$ |
| (c) $64^x = 16$ | (d) $9^{6x} = \frac{1}{27}$ |
| (e) $9^{2x+1} = 27$ | (f) $5^{2x+1} = \frac{1}{125}$ |
| (g) $32^{3x-2} = 64$ | (h) $3^{3x-1} = \frac{1}{81}$ |
| (i) $10^x = 10000$ | (j) $10^{x-2} = \frac{1}{10000}$ |
| (k) $3(5^{x+1}) = 15$ | (l) $2(3^{x-2}) = 18$ |
| (m) $5(4^x) = 10$ | (n) $3^{2x-1} + 1 = 2$ |

- (g) $\frac{1}{10000}$ (h) $\frac{1}{837}$
 2. (a) $\frac{1}{x^6}$ (b) $\frac{x^2}{y^2}$ (c) $\frac{1}{a^3b^4}$ (d) $\frac{1}{a^2}$ (e) a^{10} (f) $\left(\frac{y}{x}\right)^3$
 3. (a) x^{-3} (b) $2ab^{-4}$ (c) πx^2y
 4. (a) $\frac{8}{25}$ (b) 1024 (c) $\frac{27}{25}$ (d) 4 (e) 9 (f) 2
 (g) $\frac{5}{4}$ (h) $\frac{10}{21}$ (i) 2 (j) 625 (k) $\frac{45}{19}$ (l) $\frac{7}{4}$
 5. (a) $\frac{343x^6}{y^9}$ (b) $\frac{a^5b^{10}}{243}$ (c) $\frac{64a^3c^9}{b^6d^{12}}$ (d) $\frac{a^2}{b^3}$ (e) $a^2b^2x^2y^4$
 (f) $\frac{a^4}{b^2} - 2\frac{a^2}{b} + 1$ (g) $a^6 + a^5 - 5a^2$ (h) $2x^2 - \frac{1}{x^2} - 1$ (i) $\frac{1}{b^{2n}}$ (j) $\frac{1}{x^{2n}} - \frac{1}{y^{2m}}$

EXERCISE 1.10

1. (a) \$9658.92 (b) \$18 655.38 (c) \$7875.66 (d) \$3655.96 (e) \$3867.65
 (f) \$457.17 (g) \$6143.17 3. \$5499.49 4. \$5131.58

EXERCISE 1.11

1. (a) $\sqrt[3]{2}$ (b) $\sqrt{37}$ (c) $\sqrt[3]{x}$ (d) $\sqrt[3]{4}$ (e) $\sqrt[3]{8} = 2\sqrt{2}$
 (f) $\sqrt[4]{27}$ (g) $\sqrt[5]{a^2}$ or $(\sqrt[5]{a})^2$ (h) $\sqrt[7]{x^4}$ or $(\sqrt[7]{x})^4$ (i) $\frac{1}{\sqrt[5]{2}}$ (j) $\frac{1}{\sqrt[5]{7}}$
 (k) $\frac{1}{\sqrt[3]{a^5}}$ (l) $\sqrt[4]{81}$
 2. (a) $3^{\frac{3}{2}}$ (b) $19^{\frac{1}{2}}$ (c) $23^{\frac{1}{3}}$ (d) $x^{\frac{1}{2}}$ (e) $7^{\frac{2}{3}}$ (f) $7^{\frac{2}{3}}$
 (g) $6^{\frac{4}{5}}$ (h) $13^{\frac{5}{3}}$ (i) $a^{\frac{2}{5}}$ (j) $a^{\frac{5}{6}}$ (k) $5^{-\frac{1}{2}}$ (l) $7^{-\frac{3}{4}}$
 3. (a) 5 (b) 4 (c) 27 (d) 1 (e) $\frac{1}{6}$ (f) 21.2561
 (g) 3 (h) $\frac{1}{2}$ (i) 4 (j) $\frac{1}{8}$ (k) -2 (l) 4
 4. (a) 16 (b) 128 (c) 100 (d) $\frac{1}{4}$ (e) 27 (f) $\frac{1}{125}$
 (g) 256 (h) 3 (i) 36 (j) 343 (k) 2 (l) 162
 (m) $\frac{2}{3}$ (n) $\frac{12}{7}$ (o) $\frac{125}{512}$ (p) 96 (q) 0.08 (r) 3
 (s) 8 (t) 0 (u) 7 (v) $\frac{5}{6}$ (w) 3 (x) $\frac{5}{108}$
 5. (a) $2^{\frac{5}{8}}$ (b) $3^{\frac{8}{5}}$ (c) $x^2y^{\frac{1}{2}}$ (d) a^3b^4 (e) ab^2c^3
 (f) $x + 3x^{\frac{2}{3}}$ (g) $2x^2y^{\frac{1}{2}}$ (h) $\frac{16x^6}{y^2}$ (i) $\frac{40\sqrt{5}x^3y^{\frac{5}{2}}}{z^{\frac{3}{2}}}$ (j) $\frac{a^{\frac{9}{2}}x}{b^9y^{\frac{24}{5}}}$
 (k) $\frac{y^{\frac{1}{2}}}{x^{\frac{1}{24}}}$ (l) $a^{\frac{5}{2}}$
 6. (a) 1.319 507 91 (b) 0.172 427 286 (c) 90.597 458 (d) 2.080 083 82
 (e) 0.341 278 752 (f) 5.179 474 68
 7. (a) $3^{\frac{1}{2}}$ (b) $7^{\frac{1}{4}}$ (c) $3^{\frac{3}{2}}$ (d) $14^{-\frac{1}{3}}$

EXERCISE 1.12

1. (a) 5 (b) 3 (c) 6 (d) 2 (e) 4 (f) 2
 (g) 3 (h) 3 (i) 4 (j) 2
 2. (a) 4 (b) 3 (c) 3 (d) 16 (e) 2 (f) 4
 (g) 6 (h) 2 (i) $x \in \mathbb{N}$ (j) $x \in \mathbb{N}$, x even
 3. (a) 2 (b) -7 (c) 5 (d) 1
 (d) $x = 0, \pm 2, \pm 4, \pm 6, \dots$ (e) 2 (f) 1
 (g) 1 (h) $x \in \mathbb{I}$ (i) 4
 (j) $-\frac{5}{2}$
 4. (a) $\frac{3}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{1}{4}$ (e) $\frac{1}{4}$ (f) -2
 (g) $\frac{16}{15}$ (h) -1 (i) 4 (j) -2 (k) 0 (l) 4

Exercise 1.6

6. Write each expression as a single power.

a) $(3^2)(3^{\frac{1}{2}})$

b) $(3^2)(3^{\frac{3}{2}})$

c) $(3^2)(3^{\frac{5}{2}})$

d) $(3^2)(3^{\frac{7}{2}})$

7. Write each expression as a single power.

a) $\frac{7}{7^{\frac{1}{4}}}$

b) $\frac{7}{7^{\frac{3}{4}}}$

c) $\frac{7}{7^{\frac{5}{4}}}$

d) $\frac{7}{7^{\frac{7}{4}}}$

✓ 8. Knowledge/Understanding Write each expression as a single power.

Then evaluate the power.

a) $(5^{0.2})(5^{1.2})$

b) $(3^{2.75})(3^{1.15})$

c) $\frac{2^{8.66}}{2^{3.12}}$

d) $(7^{1.5})^{0.4}$

B

✓ 9. Solve each equation.

a) $2^{x+1} = 4$

b) $2^{x-1} = 8$

c) $3^{x-5} = 9$

d) $5^{x+3} = 25$

e) $4^{x+2} = 16$

f) $2^{2x+1} = 8$

g) $3^{2x-1} = 9$

h) $9^{1-2x} = 81$

✓ 10. a) Solve each equation.

i) $4^x = 8^{x+3}$

ii) $4^x = 8^{x+2}$

iii) $4^x = 8^{x+1}$

iv) $4^x = 8^x$

v) $4^x = 8^{x-1}$

vi) $4^x = 8^{x-2}$

vii) $4^x = 8^{x-3}$

viii) $4^x = 8^{x-4}$

b) Choose one equation from part a. Explain how you solved the equation.

✓ 11. Thinking/Inquiry/Problem Solving Consider the equation $8^{x-2} = 64^{x+1}$.

Solve the equation in two different ways.

12. Solve each equation.

a) $2^{2x} - 9(2^x) + 8 = 0$

b) $3^{2x} - 12(3^x) + 27 = 0$

c) $2^{2x} - 2(2^x) - 8 = 0$

d) $4^{2x} - 15(4^x) - 16 = 0$

e) $2^{2x} - 18(2^x) + 32 = 0$

f) $3^{2x} + 3^x - 2 = 0$

g) $3^{2x} - 6(3^x) + 9 = 0$

h) $4^{2x} - 17(4^x) + 16 = 0$

i) $4^{2x} - 16 = 0$

✓ 13. Application Scientists have measured many bird eggs. From these data, they have established approximate formulas to represent various measurements. In the two formulas below, m represents the mass of the bird in grams. The formulas give average results. The properties of a particular egg from a particular bird may differ from the calculated result.

Mass of the egg, e grams: $e = 0.277m^{0.770}$

Mass of the eggshell, s grams: $s = 0.0482e^{1.132}$

a) Use the first formula. Write an expression for the mass of the egg as a fraction of the mass of the bird.

b) Use the second formula. Write an expression for the mass of the eggshell as a fraction of the mass of the egg.

Exercise 1.6

7. a) $7^{\frac{3}{4}}$ b) $7^{\frac{1}{4}}$ c) $7^{-\frac{1}{4}}$ d) $7^{\frac{3}{4}}$

8. a) 9.52 b) 72.57 c) 46.53 d) 3.21

9. a) 1 b) 4 c) 7 d) -1
e) 0 f) 1 g) 1.5 h) -0.5

10. a) i) -9 ii) -6 iii) 0 iv) 3
v) 3 vi) 6 vii) 9 viii) 12

12. a) $0.277 m^{-0.23}$ b) $0.0482 e^{0.132}$

13. a) 0.019, 0.048, 0.206 b) 0.133, 0.088, 0.046

14. $0.0113m^{-0.128}$

Year	Population
1977	200
1980	221
1983	245
1986	271
1989	300
1992	332
1993	368
1994	500

Year	Population
1977	300
1980	378
1983	476
1986	600
1989	755
1992	952
1995	1199
1998	1510
2001	1902

17. a) i) 2.807 ii) 0.356
18. a) 19.95 b) 199.5 c) 1995 d) 19 950
e) 0.1995 f) 0.01995 g) 0.001995 h) 0.0001995

Self-Check 1.5–1.6

1. a) 1 b) 16 c) 32 d) -625
e) 1 f) $\frac{1}{2}$ g) $-\frac{1}{9}$ h) -125

2. a) 2.609 b) 15.385 c) 828.956
d) 0.0031 e) 7.622 f) -0.547

3. a) $x^{\frac{31}{20}}$ b) $x^{\frac{7}{6}}$ c) $x^{-\frac{1}{5}}$ d) $x^{\frac{1}{12}}$

4. a) 256 b) 64

5. a) 29.365 b) 166.146

6. a) 6 b) 0

1.7—The Sum of an Arithmetic Series

Exercises

1. a) 30 b) 90 c) -30 d) -40

2. a) 153 b) 385 c) 244 d) 441

3. a) 375 b) -98 c) 35 000 d) -357

4. a) 11.04 b) 55.56 c) -109.2 d) -168

5. a) 400 b) 590 c) 970 d) 115

6. a) 210 b) 320 c) -290 d) 180
e) 579 f) 37.5

7. a) 55 b) 5050
c) 500 500 d) 50 005 000

8. a) i) 416 ii) -598 iii) 3604 iv) -110

9. a) 1380 b) -637 c) 646 d) 246.5
e) -383.5 f) -137.6

10. Job A
Answers may vary. For example, one assumption could be that the salary increase occurs at the end of the 2nd pay period.

11. Job B pays more over the summer.

12. $\frac{n^2 + n}{2}$

13. a) 63 b) 690

14. 272

15. a) 44 b) 15 c) 345

16. $\frac{n}{2}[2a + (n-1)d]$

17. a) \$1520 b) \$29 885 c) Year 5 d) \$209 195

18. a) \$9000 b) \$18 000

19. -59, 38

20. 90

21. 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31

22. a) Doctors prescribe the correct pharmaceuticals, but you cannot read the prescription.
b) Each word has one more letter than the one before.
c) 210

24. a) 125 b) 15

25. $d = 2, a = 16; 16, 18, 20, 22, 24, 26, 28, 30, 32, 34$
 $d = 4, a = 7; 7, 11, 15, 19, 23, 27, 31, 35, 39, 43$