LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: __3 - 4_**

Unit/Chapter: __Exponents_& Logarithms_ **Topic: Exponential Growth and Decay_**

- ***homework check:*** **FM12** p. 219 # 1 – 7
  ASM12 p. 368 # 6 - 12

- ***note:*** **Exponential Growth and Decay**

The exponential function is used in applications of exponential growth, compound interest, investment growth, depreciation, inflation rates, and radioactive decay. The formulas $N(t) = C \cdot 2^{\frac{t}{d}}$ for growth and $N(t) = C \cdot 2^{\frac{-t}{h}}$ for decay are used to help estimate "N(t)" (the amount remaining) after time has elapsed. The "C" represents the initial amount and "t" represents the time that elapses in the experiment. It is important to note that the time elapsed and the doubling period or half life must be in the same units.

examples)

The scientist eagerly begins his experiment without an initial count. If the bacteria count is 10 000 after 6h, and he knows the doubling period is 3h, what was the initial size? What amount will be present after 1 day?

Solution: We are given: $t = 6h$, $d = 3h$, $N(6) = 10\ 000$. Find c.

$$N(t) = C \cdot 2^{\frac{t}{d}}$$
$$10000 = C \cdot 2^{\frac{6}{3}}$$
$$10000 = C \cdot 2^2$$
$$10000 = C \cdot 4$$
$$C = 2500$$

Using the above info, we can find the amount remaining after 1 day.

$$N(t) = 2500 \cdot 2^{\frac{24}{3}}$$
$$N(t) = 2500 \cdot 2^8$$
$$N(t) = 2500 \cdot 256$$
$$N(t) = 640\ 000$$

- ***homework assignment:*** **FM12** p. 223 exercise 7.2

$$\therefore \frac{1}{\sqrt{2}}c = c \times 2^{-\frac{8}{h}}$$
$$\frac{1}{\sqrt{2}} = 2^{-\frac{8}{h}}$$
$$2^{-\frac{1}{2}} = 2^{-\frac{8}{h}}$$
$$\frac{1}{2} = \frac{8}{h}$$
$$h = 16$$

The half-life of V^{48} is 16 d.

EXERCISE 7.2

A 1. Strontium-90 has a half-life of 25 a. How long would it take 4 mg of it to decay to

(a) 2 mg? (b) 1 mg?
(c) 0.25 mg? (d) $\frac{1}{2}$ mg?

2. Radium-221 has a half-life of 30 s. What fraction of a sample would remain after the following time?

(a) 30 s (b) 1 min (c) 2 min (d) 3 min

3. The doubling period of a bacteria culture is 15 min and it starts with 5000 bacteria. How many bacteria will there be after

(a) 15 min? (b) 45 min? (c) 1 h? (d) 1.5 h?

4. A bacteria culture grows according to the formula

$$N(t) = 12\,000 \times 2^{\frac{t}{4}}$$

where the time t is given in hours. How many bacteria are there

(a) at the beginning of the experiment?
(b) after 4 h?
(c) after 8 h?
(d) after 12 h?

B 5. Five million bacteria live in an organism. The doubling period is 1.5 h. How many will there be after

(a) t hours? (b) 9 h? (c) 1 d? (d) 2 h?

6. A bacteria culture doubles every $\frac{1}{4}$ h. At time $1\frac{1}{4}$ h an estimate of 40 000 is taken.

(a) What was the initial size of the culture?
(b) What is it after 2 h?
(c) What is it after 3 h?

7. A bacteria culture starts with 3000 bacteria. After 3 h the estimated count is 48 000. What is the doubling period?

8. A bacteria culture starts with 6000 bacteria. After half an hour the bacteria count is 33 600. Estimate the number of minutes it takes for the culture to double. (Use the graph of $y = 2^x$ in Section 7.1.)

9. The world population is doubling about every 35 a. In 1980 the total population was about 4.5 billion. If the doubling period remains at 35, find the projected world population for the year 2120.

10. An isotype of sodium, Na^{24}, has a half-life of 15 h.

(a) Find the amount remaining from a 4 g sample after
(i) 30 h (ii) t hours (iii) 5 d (iv) 7.5 h
(b) How many hours will it take to decay to 2^{-4} g?
(c) How many hours will it take to decay to 1.6 g?
(For (c) use the graph of $y = 2^x$.)

11. The half-life of Palladium-100, Pd^{100}, is 4 d. After 16 d a sample of Pd^{100} has been reduced to a mass of 0.75 g.

(a) What was the initial mass of the sample?
(b) What is the mass after 2 d?
(c) What is the mass after 2 weeks?

12. After 30 h a sample of Plutonium-243, Pu^{243}, has decayed to $\frac{1}{64}$ of its original mass. Find the half-life of Pu^{243}.

10. (a) (i) 1 g (ii) $4.2^{\frac{t}{15}}$ (iii) 0.015 625 g (iv) $2^{\frac{3}{2}}$ g (b) 90 h (c) 19.8 h

11. (a) 12 g (b) $6\sqrt{2} = 8.5$ g (c) 1.1 g $= 12.2^{\frac{7}{2}}$ g

12. 5 h

EXERCISE 7.3

1. (a) $\log_3 9 = 2$ (b) $\log_2 16 = 4$ (c) $\log_{16} 216 = 3$ (d) $\log_9(\frac{1}{9}) = -1$
   (e) $\log_a c = b$ (f) $\log_8 1 = 0$ (g) $\log_4 1024 = 5$ (h) $\log_{49} 7 = \frac{1}{2}$
   (i) $\log_8 4 = \frac{2}{3}$ (j) $\log_5(\frac{1}{25}) = -2$ (k) $\log_{10} 10\,000 = 4$ (l) $\log_4(0.125) = -\frac{3}{2}$
2. (a) $7^2 = 49$ (b) $3^6 = 729$ (c) $4^{4.5} = 512$ (d) $10^{-1} = 0.1$
   (e) $2^{-4} = \frac{1}{16}$ (f) $a^c = b$ (g) $12^3 = 1728$ (h) $10^0 = 1$
   (i) $5^1 = 5$ (j) $16^{0.5} = 4$ (k) $8^{\frac{2}{3}} = 4$ (l) $2^{12} = 4096$
3. (a) 2 (b) 5 (c) 3 (d) 3 (e) −1 (f) 0
   (g) −2 (h) 1 (i) 3 (j) 9 (k) 87 (l) $\sqrt{3}$
   (m) 1 (n) 19 (o) 4379 (p) $\frac{1}{2}$
4. (a) 7 (b) 4 (c) −4 (d) −3 (e) 4 (f) −2
   (g) −2 (h) −5 (i) $\frac{1}{2}$ (j) $\frac{7}{2}$ (k) $\frac{1}{4}$ (l) $-\frac{3}{2}$
5. (a) 1 000 000 (b) 256 (c) 5 (d) 5 (e) 16 (f) $\frac{1}{3}$
   (g) −1 (h) −2 (i) $\frac{1}{16}$ (j) 8 (k) 243 (l) $\frac{3}{4}$
7. (a) 1.6 (b) 1.3 (c) 2.3 (d) −0.3
10. (a) 35 (b) $\frac{7}{5}$ (c) 343 (d) 3
11. 2 L 12. M + N

(a) $\{x \mid x > -1\}$ (b) $\{x \mid x < 5\}$ (c) $\{x \mid -2 < x < 2\}$

EXERCISE 7.4

1. (a) $\log_{10} 8 + \log_{10} 13$ (b) $\log_2 9.1 + \log_2 6.3$ (c) $\log_5 14 + \log_5 8.1$
   (d) $\log_5 11 - \log_5 37$ (e) $\log_8 104 - \log_8 97.2$ (f) $\log_{10} 2 - \log_{10} \pi$
   (g) $\log_3 2 + \log_3 \pi$ (h) $\log_2 19 + \log_2 97$ (i) $\log_{12} 16 - \log_{12} 65$
   (j) $\log_{10} x + \log_{10} y$ (k) $\log_{10} x - \log_{10} y$ (l) $\log_x A + \log_x B$
2. (a) $\log_{10}(89 \times 14)$ (b) $\log_5(12.2 \times 2.79)$ (c) $\log_2(\frac{75}{36})$
   (d) $\log_3(\frac{634}{149})$ (e) $\log_6(18)$ (f) $\log_7(6)$
   (g) $\log_{10}(xy)$ (h) $\log_2\left(\frac{x}{y}\right)$ (i) $\log_{10}(9)$
   (j) $\log_9(60)$
3. (a) $2 \log_{10} 68$ (b) $5 \log_2 3.9$ (c) $10 \log_5 \pi$
   (d) $\frac{3}{4} \log_{10} 7$ (e) $\frac{1}{2} \log_3 5$ (f) $\frac{1}{2} \log_5 3$
   (g) $-\log_{10} 8$ (h) $-\log_{10} 12$ (i) $9 \log_{10} x$
   (j) $\log_{10} 37^2$ (k) $\log_2 21^8$ (l) $\log_5 2^3$
   (m) $\log_5 97^{\frac{1}{3}}$ (n) $\log_{10} 3$ (o) $\log 5^{-1}$
   (p) $\log_{10}(\frac{1}{4})$ (q) $y \log_2 x$ (r) $\log_6 A^m$
4. (a) $\log_{12} 82 + \log_{12} 28$ (b) $\log_2 9 + \log_2 13 + \log_2 14$ (c) $20 \log_5 9$
   (d) $\log_3 79 - \log_3 53$ (e) $\log_{10} 36$ (f) $\log_2 L + \log_2 M + \log_2 N$
   (g) $\log_{10} 7$ (h) $\log_2 937 - \log_2 1005$ (i) $-\log_{10} 67$
   (j) $\frac{1}{2} \log_5 83$ (k) $\log_a 5 + \log_a x$ (l) $\log_3(\frac{1}{8})$
   (m) $\log_2 42$ (n) $\log_{10} 7$
5. (a) 0.7781 (b) 1.1761 (c) 0.6020 (d) 1.2552
   (e) 2.097 (f) 0.398 (g) 0.1761 (h) −0.2219
   (i) 0.8751 (j) 0.3495 (k) 0.119275 (l) −0.3010
   (m) 2.301 (n) 4.699 (o) −2.5229 (p) 0.8662