LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: 3 - 5**

Unit/Chapter: Exponents & Logarithms **Topic: Logarithmic Functions**

□ ***homework check:*** **FM12** p. 223 exercise 7.2

□ ***note:*** **Logarithmic Functions**

Recall, the exponential function is a one-to-one function, which means that it has an inverse. The inverse of an exponential function is called the logarithmic function. Therefore, given $f(x) = a^x$ then the inverse is denoted by $f^{-1}(x) = \log_a x$ which is read as "the log of x to the base a" or log x to the base a".

We also know that in order to graph an inverse $y = f^{-1}(x)$, we reflect the graph $y = f(x)$. Therefore, we can graph the logarithmic function $f^{-1}(x) = \log_a x$ by reflecting the function $f(x) = a^x$ in the line $y = x$. Because we are reflecting the exponential function, the domain of logarithmic function is $\{x \mid x > 0,\ x \in \Re\}$.
(show basic exponential, reflection line, and basic logarithmic).

When working with exponential and logarithmic form, we need to remember the base of any exponent is the same as the base of an equivalent logarithm. Therefore, $\log_a x = y$ in logarithmic form is equivalent to $a^y = x$ in exponential form (which is the inverse form once x and y are interchanged). Notice that $\log_a x$ is the exponent to which the base must be raised to give x.

examples)

$\log_{10} 10000 = 4$ can be rewritten as $10^4 = 10000$

$4^{-1} = \frac{1}{4}$ can be rewritten as $\log_4\left(\frac{1}{4}\right) = -1$

$\log_a s = r$ can be rewritten as $a^r = s$

$2^{-3} = \frac{1}{8}$ can be rewritten as $\log_2\left(\frac{1}{8}\right) = -3$

□ ***homework assignment:*** **FM12** p. 227 exercise 7.3 # 1 – 6

EXERCISE 7.3

A 1. Express in logarithmic form.

(a) $3^2 = 9$ (b) $2^4 = 16$
(c) $6^3 = 216$ (d) $9^{-1} = \frac{1}{9}$
(e) $a^b = c$ (f) $8^0 = 1$
(g) $4^5 = 1024$ (h) $49^{\frac{1}{2}} = 7$
(i) $8^{\frac{2}{3}} = 4$ (j) $5^{-2} = \frac{1}{25}$
(k) $10^4 = 10\,000$ (l) $4^{-\frac{3}{2}} = 0.125$

2. Express in exponential form.

(a) $\log_7 49 = 2$ (b) $\log_3 729 = 6$
(c) $\log_4 512 = 4.5$ (d) $\log_{10} 0.1 = -1$
(e) $\log_2 (\frac{1}{16}) = -4$ (f) $\log_a b = c$
(g) $\log_{12} 1728 = 3$ (h) $\log_{10} 1 = 0$
(i) $\log_5 5 = 1$ (j) $\log_{16} 4 = 0.5$
(k) $\log_8 4 = \frac{2}{3}$ (l) $\log_2 4096 = 12$

3. Evaluate.

(a) $\log_2 4$ (b) $\log_2 32$
(c) $\log_{10} 1000$ (d) $\log_3 27$
(e) $\log_5 (\frac{1}{5})$ (f) $\log_9 1$
(g) $\log_6 (\frac{1}{36})$ (h) $\log_7 7$
(i) $\log_5 125$ (j) $\log_2 2^9$
(k) $\log_3 3^{87}$ (l) $\log_5 5^{\sqrt{3}}$
(m) $\log_a a$ (n) $10^{\log_{10} 19}$
(o) $a^{\log_a 4379}$ (p) $\log_x \sqrt{x}$

B 4. Evaluate.

(a) $\log_2 128$ (b) $\log_3 81$
(c) $\log_5 (\frac{1}{625})$ (d) $\log_6 (\frac{1}{216})$
(e) $\log_4 256$ (f) $\log_2 0.25$
(g) $\log_5 0.04$ (h) $\log_{10} 0.000\,01$
(i) $\log_6 \sqrt{6}$ (j) $\log_2 8\sqrt{2}$
(k) $\log_4 \sqrt{2}$ (l) $\log_4 0.125$

5. Solve the following equations for x.

(a) $\log_{10} x = 6$ (b) $\log_2 x = 8$
(c) $\log_x 25 = 2$ (d) $\log_x \frac{1}{5} = -1$
(e) $\log_x 4 = \frac{1}{2}$ (f) $\log_2 \sqrt[3]{2} = x$
(g) $\log_{\frac{1}{2}} 2 = x$ (h) $\log_{\frac{1}{3}} 9 = x$
(i) $\log_4 x = -2$ (j) $\log_x 16 = \frac{4}{3}$
(k) $\log_x 81 = \frac{4}{5}$ (l) $\log_9 3\sqrt{3} = x$

6. (a) Draw the graph of $y = 5^x$. Then reflect it in the line $y = x$ to get the graph of $y = \log_5 x$.
(b) Draw the graph of $y = \log_{10} x$ by the same method as in (a). How do the graphs of $\log_5 x$ and $\log_{10} x$ compare?

7. Use the graph of $y = 2^x$ in Section 7.1 to find approximate values for the following numbers.

(a) $\log_2 3$ (b) $\log_2 (2.5)$
(c) $\log_2 5$ (d) $\log_2 (0.8)$

8. Draw the graph of $y = |\log_2 x|$

C 9. Draw the graph of $y = \log_2 |x|$, $x \in R$, $x \neq 0$.

10. Evaluate.

(a) $10^{(\log_{10} 7 + \log_{10} 5)}$ (b) $3^{(\log_3 7 - \log_3 5)}$
(c) $8^{\log_2 7}$ (d) $2^{\log_4 9}$

11. If $\log_a x = L$, find $\log_a x^2$.

12. If $\log_a x = M$ and $\log_a y = N$, find $\log_a xy$.

13. Show that $\log_a c = \dfrac{\log_b c}{\log_b a}$.

14. Find the domains of the following functions.

(a) $f(x) = \log_2 (1 + x)$
(b) $f(x) = \log_3 (5 - x)$
(c) $f(x) = \log_5 (4 - x^2)$

MIND BENDER

Two friends set their watches at 21:00. One watch runs 2 min/h too fast and the other runs 1 min/h too slow. At what time will the faster watch be 1 h ahead of the slower watch?