LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: 3 - 6**

Unit/Chapter: Exponents & Logarithms **Topic: Laws of Logarithms**

- ***homework check:*** **FM12** p. 227 exercise 7.3 # 1 - 6

- ***note:*** **Laws of Logarithms**

Just like we have exponent laws, there are similar laws for logarithms.

The Product Law: the logarithm of a product is equal to the sum of the logarithms of the factors.

$$\log_a(MN) = \log_a M + \log_a N$$

The Quotient Law: the logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator.

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

The Power Law: the logarithm of a power of a number is equal to the exponent multiplied by the logarithm of the number. Variations might include the following

$$\log_a M^n = n\log_a M$$

$$\log_a \sqrt[n]{M} = \frac{1}{n}\log_a M$$

$$\log_a\left(\frac{1}{M}\right) = -\log_a M$$

We can use our rules to simplify and/or evaluate the individual logarithms. examples)

$$\log_3 9^2 =$$
$$= 2\log_3 9$$
$$= 2(2)$$
$$= 4$$

$$\log_2 4 + \log_2 8 =$$
$$= \log_2 32$$
$$= 5$$

- ***homework assignment:*** **FM12** p. 230 exercise 7.4 # 1 – 6, 8

1. $\log_a (MN) = \log_a M + \log_a N$

2. $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

2′. $\log_a \left(\frac{1}{N}\right) = -\log_a N$

3. $\log_a M^n = n \log_a M$

3′. $\log_a \sqrt[n]{M} = \frac{1}{n} \log_a M$

EXAMPLE 5. Express as a single logarithm.

$$\tfrac{1}{2} \log_{10} x - \log_{10} y$$

SOLUTION:
Using the Power Law and the Quotient Law, we have

$$\begin{aligned} \tfrac{1}{2} \log_{10} x - \log_{10} y &= \log_{10} x^{\frac{1}{2}} - \log_{10} y \\ &= \log_{10} \sqrt{x} - \log_{10} y \\ &= \log_{10} \frac{\sqrt{x}}{y} \end{aligned}$$

EXERCISE 7.4

A 1. Express as sums or differences of logarithms.

(a) $\log_{10} (8 \times 13)$
(b) $\log_2 (9.1 \times 6.3)$
(c) $\log_5 (14 \times 8.1)$
(d) $\log_5 (\frac{11}{37})$
(e) $\log_8 \left(\frac{104}{97.2}\right)$
(f) $\log_{10} \left(\frac{2}{\pi}\right)$
(g) $\log_3 2\pi$
(h) $\log_2 (19 \times 97)$
(i) $\log_{12} (16 \div 65)$
(j) $\log_{10} xy$
(k) $\log_{10} \frac{x}{y}$
(l) $\log_x (AB)$

2. Express as logarithms of products or quotients.

(a) $\log_{10} 89 + \log_{10} 14$
(b) $\log_5 12.2 + \log_5 2.79$
(c) $\log_2 75 - \log_2 36$
(d) $\log_3 634 - \log_3 149$
(e) $\log_6 2 + \log_6 9$
(f) $\log_7 54 - \log_7 9$
(g) $\log_{10} x + \log_{10} y$
(h) $\log_2 x - \log_2 y$
(i) $\log_{10} 36 - \log_{10} 4$
(j) $\log_9 12 + \log_9 5$

3. Apply the Power Law to the following.

(a) $\log_{10} 68^2$
(b) $\log_2 3.9^5$
(c) $\log_5 \pi^{10}$
(d) $\log_{10} 7^{\frac{3}{4}}$
(e) $\log_3 5^{\frac{1}{2}}$
(f) $\log_5 \sqrt{3}$
(g) $\log_{10} 8^{-1}$
(h) $\log_{10} (\frac{1}{12})$
(i) $\log_{10} x^9$
(j) $2 \log_{10} 37$
(k) $8 \log_2 21$
(l) $3 \log_5 2$
(m) $\frac{1}{3} \log_5 97$
(n) $\frac{1}{2} \log_{10} 9$
(o) $-\log_{10} 5$
(p) $-\frac{1}{2} \log_{10} 16$
(q) $\log_2 x^y$
(r) $m \log_6 A$

4. Apply the Laws of Logarithms to the following.

(a) $\log_{12}(82 \times 28)$ (b) $\log_2(9 \times 13 \times 14)$
(c) $\log_5 9^{20}$ (d) $\log_3(79 \div 53)$
(e) $2 \log_{10} 6$ (f) $\log_2(LMN)$
(g) $\frac{1}{2} \log_{10} 49$ (h) $\log_2(\frac{937}{1005})$
(i) $\log_{10}(\frac{1}{67})$ (j) $\log_5 \sqrt{83}$
(k) $\log_a(5x)$ (l) $-\log_3 8$
(m) $\log_2 6 + \log_2 7$ (n) $\log_{10} 28 - \log_{10} 4$

B 5. Given the approximate values $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, and $\log_{10} 5 = 0.6990$, evaluate the following.

(a) $\log_{10} 6$ (b) $\log_{10} 15$
(c) $\log_{10} 4$ (d) $\log_{10} 18$
(e) $\log_{10} 125$ (f) $\log_{10}(\frac{5}{2})$
(g) $\log_{10} 1.5$ (h) $\log_{10}(\frac{3}{5})$
(i) $\log_{10} 7.5$ (j) $\log_{10} \sqrt{5}$
(k) $\log_{10} \sqrt[4]{3}$ (l) $\log_{10} \frac{1}{2}$
(m) $\log_{10} 200$ (n) $\log_{10} 50\,000$
(o) $\log_{10} 0.003$ (p) $\log_{10} \sqrt{54}$

6. Use the Laws of Logarithms to evaluate the following.

(a) $\log_4 2 + \log_4 32$ (b) $\log_{10} 1.25 + \log_{10} 80$
(c) $\log_3 108 - \log_3 4$ (d) $\log_2 80 - \log_2 5$
(e) $\log_{12} 16 + \log_{12} 9$ (f) $\log_3 \sqrt[3]{9}$
(g) $\log_2 8^{27}$ (h) $\log_{10} \sqrt{0.1}$
(i) $\log_8 6 - \log_8 3 + \log_8 2$ (j) $\log_5 5\sqrt{5}$

7. (a) Use your calculator to draw carefully the graph of $y = \log_{10} x$ for $0.1 \leq x \leq 10$.
(b) Find approximate values for $\log_{10} 2$, $\log_{10} 3$, and $\log_{10} 6$ from your graph. Use these values to illustrate the Product Law.
(c) Find approximate values for $\log_{10} 9$ and $\log_{10} 2$ from your graph. Use the Quotient Law to predict the value of $\log_{10} 4.5$ and compare with the value from your graph.
(d) Find an approximate value for $\log_{10} 2.5$ from your graph. Use the Power Law to predict the value of $\log_{10} 6.25$ and compare with the value from your graph.

8. Express as a single logarithm.

(a) $\log_3 6 + 4 \log_3 2$
(b) $\log_6 3 + \frac{1}{2} \log_6 5 - \log_6 2$
(c) $\log_2 a + \log_2 b - \log_2 c$
(d) $\log_{10} a + \frac{1}{2} \log_{10} b - 2 \log_{10} c$
(e) $\frac{1}{2}[\log_{10} x + \log_{10} y] - 2 \log_{10} c$
(f) $\frac{1}{2}[(\log_5 a + 2 \log_5 b) - 3 \log_5 c]$
(g) $\log_2(a + b) + \log_2(a - b) - 2 \log_2 a$
(h) $\log_2 a + b \log_2 c - d \log_2 e$

C 9. Find the error.

$$\log_3 0.1 < 2 \log_3 0.1$$
$$= \log_3 (0.1)^2$$
$$= \log_3 0.01$$
$$\log_3 0.1 < \log_3 0.01$$
$$\therefore 0.1 < 0.01$$

JOHN NAPIER (1550–1617)

John Napier was a Scottish lord, the Baron of Merchiston, who spent most of his time managing his estates and writing books on a great variety of topics, especially religion. As an amateur mathematician, he was mainly interested in computation and trigonometry. He started to work on his greatest achievement, the invention of logarithms, in about 1594, but it was not until 1614 that he published a book containing the first table of logarithms. This book was immediately hailed as a great labour-saving device by mathematicians, astronomers, and other scientists.