LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: __3 - 7__**

Unit/Chapter: __Exponents & Logarithms__ **Topic: __Using Logarithms__**

- ***homework check:*** **__FM12__** p. 230 exercise 7.4 # 1 – 6, 8

- ***note:*** **__Using Logarithms to Solve Equations__**

Logarithms are used expressly to solve any exponential equation which cannot be reduced to common bases. Recall that in basic equation solving, we systematically either add or subtract the same amount on both sides, or we multiply or divide by the same amounts on both sides. Similarly, we can take the log to the same base of both sides to introduce logarithms enabling us to solve an equation. For example,

$$
\begin{aligned}
3^{x+2} &= 4 \\
\log_3 3^{x+2} &= \log_3 4 \\
(x+2)\log_3 3 &= \log_3 4 \\
x+2 &= \log_3 4 \\
x &= \log_3 4 - 2
\end{aligned}
$$

(exact answer)

If we want to evaluate the answer to the indicated number of decimal places, we need to know how to change bases to the common base 10 that our calculators use.

$$
\begin{aligned}
\log_{10} 3^{x+2} &= \log_{10} 4 \\
(x+2)\log_{10} 3 &= \log_{10} 4 \\
x+2 &= \frac{\log 4}{\log 3} \\
x &= \frac{\log 4}{\log 3} - 2 \\
x &\doteq -0.7381
\end{aligned}
$$

In general, we can derive this notion given $y = \log_b x$ which is exponentially equivalent to $b^y = x$. Given $b^y = x$, take the log of each side to the newly required base, called “a”

$$
\begin{aligned}
\log_a b^y &= \log_a x \\
y \log_a b &= \log_a x
\end{aligned}
$$

But recall $y = \log_b x$, so we can substitute back to get

$$\log_b x(\log_a b) = \log_a x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

The knowledge of changing bases is helpful when asked to find the answer in decimal form. Knowing the general application, we can go back to our first example to see the change in action. Therefore, $x = \log_3 4 - 2$ and $x = \dfrac{\log 4}{\log 3} - 2$ are equivalent expressions in exact form. Also note that because 10 is the common base, we do not need to write it.

⌗ ***homework assignment:*** **<u>FM12</u>** p. 233 exercise 7.5 # 1 – 4

SOLUTION:

$$\begin{aligned} \log_{10}(x+2) + \log_{10}(x-1) &= 1 \\ \log_{10}(x+2)(x-1) &= \log_{10} 10 \\ \therefore (x+2)(x-1) &= 10 \\ x^2 + x - 2 &= 10 \\ x^2 + x - 12 &= 0 \\ (x+4)(x-3) &= 0 \\ x = -4 \quad \text{or} \quad x &= 3 \end{aligned}$$

If $x = -4$, then $\log(x + 2) = \log(-4 + 2) = \log(-2)$ is not defined. Hence the root $x = -4$ is inadmissible.
$\therefore$ The only root is $x = 3$.

EXERCISE 7.5

B 1. Solve each of the following equations exactly.

(a) $2^x = 5$ (b) $3^x = 10$
(c) $10^{x-4} = 7$ (d) $5^{1-x} = 2$
(e) $4^{2x} = 15$ (f) $6^{\frac{x}{3}} = 29$

2. Find the roots of the following equations correct to 4 decimal places.

(a) $10^x = 16$ (b) $7^x = 43$
(c) $2^{-x} = 6$ (d) $3^{1+x} = 36$
(e) $4^{3x} = 21$ (f) $8^{-\frac{x}{3}} = 20$
(g) $5^{2x+3} = 30$ (h) $2^{x^2} = 10$

3. Solve for x.

(a) $\log_2 x = \log_2 5 + \log_2 3$
(b) $\log_2 x = \log_2 18 - \log_2 6$
(c) $\log_{10} x + \log_{10} 12 = \log_{10} 8$
(d) $\log_{10} x = 1 + \log_{10} 2$
(e) $\log_3 x + \log_3 (x - 1) = \log_3 (2x)$
(f) $\log_9 (x - 5) + \log_9 (x + 3) = 1$
(g) $\log_2 (x + 1) - \log_2 (x - 1) = 1$
(h) $3 \log_2 x = \log_2 8$
(i) $\log_{10} x = 3 \log_{10} 7$
(j) $4 \log_6 x = \log_6 625$

C 4. Solve for x.

(a) $\log_2 (3x + 2) - \log_2 (x - 2) = 3$
(b) $\log_{10} (1 + \sqrt{x}) = 1 + \log_{10} (1 - \sqrt{x})$

EXTRA

Scientists can determine the age of certain objects by a method called radiocarbon dating. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to Carbon 14 with a half-life of 5760 a. This C 14 is assimilated by all plants and animals. When the plant or animal dies it cannot assimilate new C 14, and the amount present at death decreases by radioactivity as time passes.

EXERCISE

1. A small sample of a bone was burned and the resulting carbon dioxide was tested with a Geiger counter. It was found that the amount of C 14 had decayed to $\frac{1}{8}$ of its original amount. How old was the bone?

(i) $\frac{\ }{3}$ (g) 0

8. (a) $\log_3(6 \times 2^4)$ (b) $\log_6\left(\frac{3 \times \sqrt{5}}{2}\right)$ (c) $\log_2\left(\frac{ab}{c}\right)$ (d) $\log_{10}\left(\frac{a\sqrt{b}}{c^2}\right)$
(e) $\log_{10}\left(\frac{\sqrt{xy}}{c^2}\right)$ (f) $\log_5\left(\sqrt{\frac{ab^2}{c^3}}\right)$ (g) $\log_2\left(\frac{a^2 - b^2}{a^2}\right)$ (h) $\log_2\left(\frac{ac^b}{e^d}\right)$

9. $\log_3 0.1$ is negative, so $2 \log_3 0.1 < \log_3 0.1$

EXERCISE 7.5

1. (a) $x = \log_2 5$ (b) $x = \log_3 10$ (c) $x = \log_{10} 7 + 4$
(d) $x = 1 - \log_5 2$ (e) $x = \frac{1}{2}\log_4 15$ (f) $x = 3 \log_6 29$
2. (a) 1.2041 (b) 1.9328 (c) −2.5849 (d) 2.2618
(e) 0.7320 (f) −4.3219 (g) −0.4433 (h) ±1.8226
3. (a) $x = 15$ (b) 3 (c) $x = \frac{2}{3}$ (c) $x = 20$ (e) $x = 3$
(f) $x = 6$ (g) 3 (h) $x = 2$ (i) $x = 7^3 = 343$ (j) $x = 5$
4. (a) $x = \frac{18}{5}$ (b) $x = \frac{81}{121}$

EXERCISE 7.6

1. (a) 10 times as strong (b) 10 times as strong (c) 100 times as strong
2. (a) 1000 times as loud (b) 10 000 000 times as loud (c) 10 000 000 000 times as loud
3. 83.04 a
4. 129.6 s
5. 149.1 h
6. 6.643 h
7. 103.3 min
8. 45.38 min
9. 2.5 a
10. 9.749%
11. 3162 times as intense
12. 3981 times louder
13. $x = b + \frac{b(b - a)}{a2^{\frac{1}{c}} - b}$

EXERCISE 7.7

1. (a) $x = \frac{-1 \pm \sqrt{101}}{2}$ (b) $x = \frac{-3 + \sqrt{521}}{4}$
(c) $x = \frac{10 \pm \sqrt{19}}{9}$ (d) $x = 100$ or $x = \frac{1}{10\,000}$
2. (a) $x = \frac{-5 \pm \sqrt{37}}{2}$ (b) $x = 1 \pm \sqrt{3}$ (c) $x = \frac{1 \pm \sqrt{1 + 4\log_{10}2}}{2}$
3. (a) $x = 81$ (b) $x = 512$ (c) $x = \log_3 4$
(d) $x = \log_5(\log_3 4)$ (e) $x = \log_4(\log_3(\log_2 5))$
4. $f^{-1}(x) = \log_2\left(\frac{x}{1 - x}\right)$, domain $\{x \mid 0 < x < 1\}$
5. $n = 5$

7.8 REVIEW EXERCISE

1. (a) 223.8721 (b) 0.1686 (c) 713.2162 (d) 0.7429
(e) 0.0574 (f) 0.3633 (g) 0.3010 (h) 4.5045
2. (a) 2 (b) 4 (c) 3 (d) 3
(e) 0 (f) −1 (g) $\frac{1}{2}$ (h) 7
3. (a) $\log_{10}14 + \log_{10}29$ (b) $\log_2 11 + \log_2 13 + \log_2 15$
(c) $10 \log_3 2$ (d) $\log_{10}61 - \log_{10}43$
(e) $\frac{1}{2}\log_5 37$ (f) $-\log_{10}6$
(g) $\log_2 3 + \log_2 \pi$ (h) $\log_2 5$