LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson : __3 - 8_**

Unit/Chapter: __Exponents_& Logarithms_ **Topic: Applications of Logarithms**

- ***homework check:*** **FM12** p. 233 exercise 7.5 # 1 – 3

- ***note:*** **Applications of Logarithms**

Logarithms can be used to help us solve any exponential equation in which we cannot Get a common base. Therefore, any half-life or exponential growth scenario could possibly introduce logarithms. For example,

The half-life of radium – 226 if 1620 a. Starting with a sample of 120 mg, after how many years is only 40 mg remaining?

$$M(t) = c \cdot 2^{\frac{-t}{h}}$$

$$40 = 120 \cdot 2^{\frac{-t}{1620}}$$

$$\frac{1}{3} = 2^{\frac{-t}{1620}}$$

$$\log\left(\frac{1}{3}\right) = \log 2^{\frac{-t}{1620}}$$

$$-\log 3 = \left(\frac{-t}{1620}\right)\log 2$$

$$1620\log 3 = t\log 2$$

$$t = \frac{1620\log 3}{\log 2}$$

$$t \doteq 2568$$

Therefore, the sample is reduced to 40 mg in 2568 years.

- ***homework assignment:*** **FM12** exercise 7.6 # 3 – 8 (bonus for #13)
exercise 7.7 # 1 – 3

EXERCISE 7.6

A 1. Three earthquakes occurred in locations A, B, and C with magnitudes 3, 4, and 5 respectively on the Richter scale. How many times stronger was the earthquake
(a) at B than the earthquake at A?
(b) at C than the earthquake at B?
(c) at C than the earthquake at A?

2. According to the table on the preceding page, amplified rock music has a loudness of 120 dB, ordinary conversation has a loudness of 50 dB, and whispering has a loudness of 20 dB. How many times louder is
(a) ordinary conversation than whispering?
(b) rock music than ordinary conversation?
(c) rock music than whispers?

B In questions 3 to 9 give your answer correct to 4 significant digits.

3. Strontium-90 has a half-life of 25 a. How many years does it take for a 20 mg sample to decay to a mass of 2 mg?

4. Radium-221 has a half-life of 30 s. How long will it take for 95% of it to decompose?

5. If 25 mg of a radioactive element decays to 20 mg in 48 h, find the half-life of the element.

6. If, under certain conditions, the number of bacteria in a jug of milk doubles in one hour, in how many hours will it be 100 times the original number?

7. A bacteria culture starts with 100 000 bacteria and the doubling period is 40 min. After how many minutes will there be 600 000 bacteria?

8. A bacteria culture starts with 50 000 bacteria. After 60 min the count is 125 000. What is the doubling period?

9. Julia Peterson wants to invest $3000 in savings certificates which bear an interest rate of 9.25% compounded semi-annually. How long a time period should she choose in order to save an amount of $3700?

10. A sum of $1000 was invested for four years and the interest was compounded semi-annually. If this sum amounts to $1,463.44 after the four years, what was the interest rate?

11. The 1906 earthquake in San Francisco had a magnitude of 8.3 on the Richter scale. At the same time in Japan there was an earthquake of magnitude 4.8 which caused only minor damage. How many times more intense was the San Francisco earthquake than the Japan earthquake?

12. A power mower makes a noise which is measured at 106 dB. Ordinary traffic registers about 70 dB. How many times louder is the mower than the traffic?

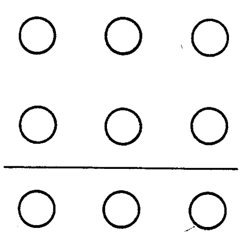
C 13. In chemistry the equation

$$t = c \log_2 \frac{b(a - x)}{a(b - x)}$$

is used where x is the concentration of a substance at time t and a, b, c are constants. Solve this equation to express x as a function of t.

MIND BENDER

Place the digits from 1 to 9 in the circles to make a correct addition sum. No two consecutive digits must be in adjacent circles.



One aspect of problem solving is that the solution of a given problem may require methods from two or more different areas of mathematics. In the following example we have to combine knowledge of logarithms with the use of the quadratic formula.

EXAMPLE. Solve the equation. $\log_{10} x + \log_{10}(x - 1) = 1$

SOLUTION:

$$\begin{aligned} \log_{10} x + \log_{10}(x - 1) &= 1 \\ \log_{10} x(x - 1) &= 1 \\ x(x - 1) &= 10 \\ x^2 - x - 10 &= 0 \end{aligned}$$

This is a quadratic equation with $a = 1$, $b = -1$, and $c = -10$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-10)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{41}}{2}$$

But $x > 0$, since $\log_{10} x$ is not defined for negative values of x. Thus, we reject the minus sign and the only solution is $x = \frac{1 + \sqrt{41}}{2}$.

EXERCISE 7.7

B 1. Solve the following equations.

(a) $\log_5(x^2 - x) = 2$
(b) $\log_2 x + \log_2(2x + 3) = 6$
(c) $\log_3(20x) = 2 + \log_3(x^2 + 1)$
(d) $\log_{10}(x^{\log_{10} x}) + \log_{10}(x^2) = 8$

2. Solve the following equations.

(a) $2^{x^2+5x} = 8$
(b) $3^{x^2} = 9^{x+1}$
(c) $10^{x^2-x} = 2$

3. Solve for x.

(a) $\log_2(\log_3 x) = 2$
(b) $\log_4(\log_3(\log_2 x)) = \frac{1}{2}$
(c) $2^{3^x} = 16$
(d) $3^{5^x} = 4$
(e) $2^{3^{4^x}} = 5$

4. Find the inverse of the function $f(x) = \frac{2^x}{1 + 2^x}$. What is the domain of the inverse function?

C5. A sequence is given by

$$3, 3^2, 3^3, 3^4, \cdots, 3^n, \cdots$$

Find the smallest value of n such that the product of the first n terms is greater than a million.

6. Show that if $x > 0$ and $x \neq 1$, then

$$\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} = \frac{1}{\log_{60} x}.$$