LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: 6 - 3**

Unit/Chapter: Other Function Types **Topic: Absolute Equations**

□ ***homework check:*** **HRW** exercise 1.3 p. 29 omit # 19 – 24, 38
exercise 1.5 p. 63 omit # 19 – 22, 35 – 47

□ ***note:*** **Absolute Equations**

We have had experience with the many different types of functions, but we have not yet talked about solving the many different types of equations. Specifically, we need to discuss absolute and radical equations.

In order to solve absolute equations, we need to rely on the definition of absolute value, meaning that what was in the absolute brackets was either positive or negative before applying the absolute value.

example) Solve $|x+4| = 5x-2$

Therefore, we use the definition to get:

$$x+4 = 5x-2 \qquad -(x+4) = 5x-2$$
$$-4x = -6 \quad \text{or} \quad -6x = 2$$
$$x = \frac{3}{2} \qquad x = \frac{-1}{3}$$

Notice that an absolute value equation yields two solutions, much like a quadratic. If we had a quadratic held within the absolute value bars, there would be four possible solutions.

□ ***homework assignment:*** **HRW** exercise 2.4 p. 116 # 9 – 20

Exercises 2.4

In Exercises 1–8, rewrite each statement using the geometric definition of absolute value. Represent each on a number line, and find the value(s) of the variable.

1. The distance between y and 2 is 4.
2. The distance between x and 4 is 6.
3. The distance between $3w$ and 2 is 8.
4. The distance between $4x$ and 3 is 6.
5. The distance between $-2x$ and 4 is 5.
6. The distance between $-4z$ and 3 is 11.
7. The distance between $-3x$ and -2 is 5.
8. The distance between $-4w$ and -6 is $\frac{5}{2}$.

In Exercises 9–20, find all real solutions of each equation.

9. $|2x + 3| = 9$
10. $|3x - 5| = 7$
11. $|6x - 9| = 0$
12. $|4x - 5| = -9$
13. $|2x + 3| = 4x - 1$
14. $|3x - 2| = 5x + 4$
15. $|x - 3| = x$
16. $|2x - 1| = 2x + 1$
17. $|x^2 + 4x - 1| = 4$
18. $|x^2 + 2x - 9| = 6$
19. $|x^2 - 5x + 1| = 3$
20. $|12x^2 + 5x - 7| = 4$
21. Explain why there are no real numbers that satisfy the equation $|2x^2 + 3x| = -12$.
22. Describe in words the meaning of the inequality.

$$|a + b| \leq |a| + |b|$$

Make sure to consider positive and negative values of a and b.

23. Joan weighs 120 pounds and her doctor told her that her weight is 5 percent from her ideal weight. What are the possible values, to the nearest pound, for Joan's ideal body weight?
24. A tightrope walker is 8 feet from one end of the rope. If he takes 2 steps and each step is 10 inches long, how far is he from the same end of the rope? Give both possible answers.
25. An instrument measures a wind speed of 20 feet per second. The true wind speed is within 5 feet per second of the measured wind speed. What are the possible values for the true wind speed?
26. For two real numbers s and t, the notation $\min(s, t)$ represents the smaller of the two numbers. When $s = t$, $\min(s, t)$ represents the common value. It can be shown that $\min(s, t)$ can be expressed as shown.

$$\min(s, t) = \frac{s + t - |s - t|}{2}$$

For each of the following, verify the equation.
a. $s = 4$ and $t = 1$
b. $s = -2$ and $t = 3$
c. $s = t = -5$

27. In statistical quality control, one needs to find the proportion of the product that is not acceptable. The upper and lower control limits (CL) are found by solving the following equation in which p is the mean percent defective, and n is the sample size for CL.

$$|CL - p| = 3\sqrt{\frac{p(1 - p)}{n}}$$

Find CL when $p = 0.02$ and $n = 200$.

In Exercises 28–63, find all real solutions of each equation. Find exact solutions when possible, approximate solutions otherwise.

28. $\sqrt{x - 7} = 4$
29. $\sqrt{4x + 9} = 5$
30. $\sqrt{3x - 2} = 7$
31. $\sqrt[3]{5 - 11x} = 3$
32. $\sqrt[3]{6x - 10} = 2$
33. $\sqrt[3]{x^2 - 1} = 2$
34. $\sqrt[3]{(x + 1)^2} = 4$
35. $\sqrt{x^2 - x - 1} = 1$
36. $\sqrt{x^2 - 5x + 4} = 2$
37. $\sqrt{x + 7} = x - 5$
38. $\sqrt{x + 5} = x - 1$
39. $\sqrt{3x^2 + 7x - 2} = x + 1$
40. $\sqrt{4x^2 - 10x + 5} = x - 3$
41. $\sqrt[3]{x^3 + x^2 - 4x + 5} = x + 1$
42. $\sqrt[3]{x^3 - 6x^2 + 2x + 3} = x - 1$
43. $\sqrt[5]{9 - x^2} = x^2 + 1$

which the radius is to be reduced. Then $r = 8 - x$ and the new area is $\pi(8 - x)^2$, which must be 48π less than the original area, that is, $\pi(8 - x)^2 = 64\pi - 48\pi$, or equivalently, $\pi(8 - x)^2 = 16\pi$.

9. \$366.67 at 12% and \$733.33 at 6%

11. $2\frac{2}{3}$ qt 13. 65 mph

15. 34.75 and 48 17. Approximately 1.75 ft

19. Red Riding Hood, 54 mph; wolf, 48 mph

21. a. ≈6.3 sec b. ≈4.9 sec

23. a. Approximately 4.4 sec b. After 50 sec

25. 23 cm by 24 cm by 25 cm 27. $r = 4.658$

29. $x = 2.234$ 31. 2.2 by 4.4 by 4 ft high

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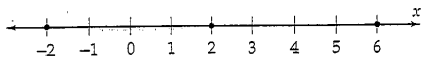
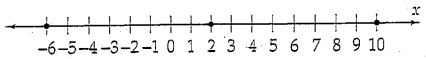
1. $|y - 2| = 4$



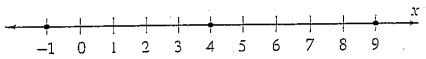
$y = -2, 6$

3. $|3w - 2| = 8$



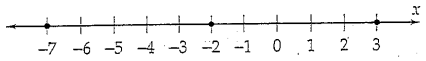
Since $3w = -6$ or 10, dividing by 3, $w = -2, 3\frac{1}{3}$.

5. $|-2x - 4| = 5$



Since $-2x = -1$ or 9, dividing by -2, $x = \frac{1}{2}, -\frac{9}{2}$.

7. $|-3x - (-2)| = 5$ or $|-3x + 2| = 5$



Since $-3x = -7$ or 3, dividing by -3, $x = \frac{7}{3}, -1$.

9. $x = -6$ or 3 11. $x = \frac{3}{2}$ 13. $x = 2$

15. $x = \frac{3}{2}$ 17. $x = -5$ or 1 or -3 or -1

19. $x = 1$ or 4 or $\frac{5 + \sqrt{33}}{2}$ or $\frac{5 - \sqrt{33}}{2}$

21. For any real number x, the distance between $2x^2$ and $-3x$ cannot be a negative number. Therefore, $|2x^2 + 3x| = -12$ has no real number solution.

23. Let x = Joan's ideal body weight. The difference between Joan's actual weight and her ideal weight is $x - 120$.

Therefore, $|x - 120| = 0.05x$.

$x - 120 = 0.05x$ or $x - 120 = -0.05x$

$0.95x = 120$ $1.05x = 120$

$x \approx 126.3$ $x \approx 114.3$

To-the nearest pound, Joan's ideal weight is either 114 pounds or 126 pounds.

25. If the true wind speed differs by 5 feet per second from the measured speed, let x = true wind speed and $|x - 20| = 5$

$x - 20 = -5$ or $x - 20 = 5$

$x = 15$ or $x = 25$

The true wind speed is between 15 and 25 feet per second.

27. CL ≈ −0.0097 = 0 (in practice) and ≈ 0.0497

29. $x = 4$ 31. $x = -2$ 33. $x = \pm 3$

35. $x = -1$ or 2 37. $x = 9$ 39. $x = \frac{1}{2}$

41. $x = \frac{1}{2}$ or -4 43. $x \approx \pm 0.73$

45. $x \approx -1.17$ or 2.59 or $x = -1$ 47. $x \approx 1.658$

49. $x = 6$ 51. $x = 3$ or 7 53. No solutions

55. $x \approx -0.457$ or 1.40 57. $x = 7$

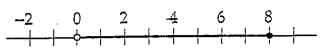
59. $x = \frac{3 \pm \sqrt{41}}{4}$ 61. $x = 1$ 63. $x = -1$

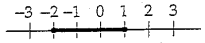
65. $u = \sqrt{\frac{x^2}{1 - K^2}}$ 67. $b = \sqrt{\frac{a^2}{A^2 - 1}}$

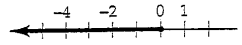
69. a. $I = \frac{x}{(x^2 + 1024)^{\frac{3}{2}}}$ b. 22.63 ft

Section 2.5, page 124

1. $x \geq 0$ 3. $c \leq 3$

5. 

7. 

9. 

11. [5, 8] 13. (−3, 14) 15. $[-8, \infty)$

17. $\left(-\infty, \frac{3}{2}\right]$ 19. $(-2, \infty)$ 21. $\left(-\infty, -\frac{8}{5}\right]$

23. $(1, \infty)$ 25. (2, 4) 27. $\left[-3, \frac{5}{2}\right)$

29. $\left(-\infty, \frac{4}{7}\right)$ 31. $\left[-\frac{7}{17}, \infty\right)$ 33. $\left[-1, \frac{1}{8}\right)$

35. $[5, \infty)$ 37. $x < \frac{b + c}{a}$

39. $c < x < a + c$ 41. $1 \leq x \leq 3$

Selected Answers