LESSON PLAN

Course: Grade 12 U Advanced Functions **Lesson: 6 - 5**

Unit/Chapter: Other Function Types **Topic: Solving Inequalities**

- ***homework check:*** **HRW** exercise 2.4 p. 116

- ***note:*** **Solving Inequalities**

Remember the basic ideas in working with inequalities. All regular equation solving rules apply with the addition of a new one – whenever multiplying or dividing by a negative number, the direction of the inequality reverses. For example,

$$-4(x+3) > -6x$$
$$-4x - 12 > -6x$$
$$-12 > -2x$$
$$\frac{-12}{-2} < \frac{-2x}{-2}$$
$$6 < x$$

To solve an extended inequality like $2 \le 3x + 5 < 2x + 11$, means that you have a solution to both parts of the inequality

$2 \le 3x + 5$		$3x + 5 < 2x + 11$
$-3 \le 3x$	and	$x + 5 < 11$
$-1 \le x$		$x < 6$

Therefore the solution set of this extended inequality is $-1 \le x < 6$ which can be written interval notation as $[-1, 6)$. Notice that in interval notation, the square bracket represent the less than and equal, while a curved bracket represents only the less than sign. This particular method works well when there are x's in more than one part of the inequality.

When there are only x's in one portion of the extended inequality, we can work quickly to add/subtract/multiply or divide to all three portions of the inequality to solve. For example,

$4 < 3 - 5x < 18$	begin by subtracting 3 from each portion
$1 < -5x < 15$	now divide each portion by -5, changing the direction of the inequalities
$\frac{-1}{5} > x > -3$	and in interval notation, $\left(-3, \frac{-1}{5}\right)$

The answer for any given extended inequality will always be the intersection of the solutions yielded. (Note to self – be sure that any questions made up always have sign facing in same direction.)

- ***homework assignment:*** **HRW** exercise 2.5 p. 124 # 11 – 47

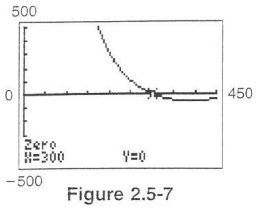


Figure 2.5-7

In this context, the only solutions that make sense are those between 0 and 450. Therefore, choose a viewing window, such as the one shown in Figure 2.5-7, and graph

$$f(x) = 2x + \frac{300{,}000}{x} - 1600.$$

The graph in Figure 2.5-7 shows that the zero of f is $x \approx 300$, and the graph of C is negative, i.e., below the x-axis, for values greater than 300. Therefore, to keep costs under \$1600, between 300 and 450 printers should be ordered per delivery. ■

Exercises 2.5

In Exercises 1–4, express the given statement in symbols.

1. x is nonnegative.
2. t is positive.
3. c is at most 3.
4. z is at least -17.

In Exercises 5–10, represent the given interval on a number line.

5. $(0, 8]$
6. $(0, \infty)$
7. $[-2, 1]$
8. $(-1, 1)$
9. $(-\infty, 0]$
10. $[-2, 7)$

In Exercises 11–16, use interval notation to denote the set of all real numbers x that satisfy the given inequality.

11. $5 \le x \le 8$
12. $-2 \le x \le 7$
13. $-3 < x < 14$
14. $7 < x < 135$
15. $x \ge -8$
16. $x \ge 12$

In Exercises 17–36, solve the inequality and express your answer in interval notation.

17. $2x + 4 \le 7$
18. $3x - 5 > -6$
19. $3 - 5x < 13$
20. $2 - 3x < 11$
21. $6x + 3 \le x - 5$
22. $5x + 3 \le 2x + 7$
23. $5 - 7x < 2x - 4$
24. $5 - 3x > 7x - 3$
25. $2 < 3x - 4 < 8$
26. $1 < 5x + 6 < 9$
27. $0 < 5 - 2x \le 11$
28. $-4 \le 7 - 3x < 0$
29. $2x + 7(3x - 2) < 2(x - 1)$
30. $x + 3(x - 5) \ge 3x + 2(x + 1)$
31. $\frac{x+1}{2} - 3x \le \frac{x+5}{3}$
32. $\frac{x-1}{4} + 2x \ge \frac{2x-1}{3} + 2$
33. $2x + 3 \le 5x + 6 < -3x + 7$
34. $4x - 2 < x + 8 < 9x + 1$
35. $3 - x < 2x + 1 \le 3x - 4$
36. $2x + 5 \le 4 - 3x < 1 - 4x$

In Exercises 37–40, the constants a, b, c, and d are positive. Solve each inequality for x.

37. $ax - b < c$
38. $d - cx > a$
39. $0 < x - c < a$
40. $-d < x - c < d$

In Exercises 41–70, solve the inequality. Find exact solutions when possible, and approximate them otherwise.

41. $x^2 - 4x + 3 \le 0$
42. $x^2 - 7x + 10 \le 0$
43. $x^2 + 9x + 15 \ge 0$
44. $x^2 + 8x + 20 \ge 0$
45. $8 + x - x^2 \le 0$
46. $4 - 3x - x^2 \ge 0$
47. $x^3 - x \ge 0$
48. $x^3 + 2x^2 + x > 0$
49. $x^3 - 2x^2 - 3x < 0$
50. $x^4 - 14x^3 + 48x^2 \ge 0$

51. $x^4 - 5x^2 + 4 < 0$

52. $x^4 - 10x^2 + 9 \le 0$

53. $x^3 - 2x^2 - 5x + 7 \ge 2x + 1$

54. $x^4 - 6x^3 + 2x^2 < 5x - 2$

55. $2x^4 + 3x^3 < 2x^2 + 4x - 2$

56. $x^5 + 5x^4 > 4x^3 - 3x^2 + 2$

57. $\dfrac{3x+1}{2x-4} > 0$

58. $\dfrac{2x-1}{5x+3} \ge 0$

59. $\dfrac{x^2+x-2}{x^2-2x-3} < 0$

60. $\dfrac{2x^2+x-1}{x^2-4x+4} \ge 0$

61. $\dfrac{x-2}{x-1} < 1$

62. $\dfrac{-x+5}{2x+3} \ge 2$

63. $\dfrac{x-3}{x+3} \le 5$

64. $\dfrac{2x+1}{x-4} > 3$

65. $\dfrac{2}{x+3} \ge \dfrac{1}{x-1}$

66. $\dfrac{1}{x-1} < \dfrac{-1}{x+2}$

67. $\dfrac{x^3 - 3x^2 + 5x - 29}{x^2 - 7} > 3$

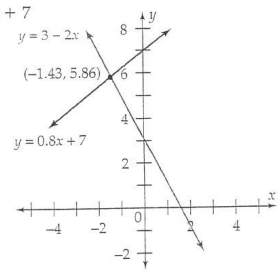
68. $\dfrac{x^4 - 3x^3 + 2x^2 + 2}{x - 2} > 15$

69. $\dfrac{2x^2 + 6x - 8}{2x^2 + 5x - 3} < 1$ Be alert for hidden behavior.

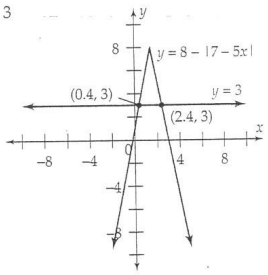
70. $\dfrac{1}{x^2 + x - 6} + \dfrac{x-2}{x+3} > \dfrac{x+3}{x-2}$

In Exercises 71–73, read the solution of the inequality from the given graph.

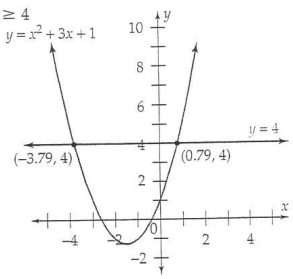
71. $3 - 2x < 0.8x + 7$



72. $8 - |7 - 5x| > 3$



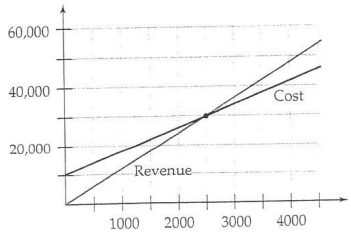
73. $x^2 + 3x + 1 \ge 4$



74. The graphs of the revenue and cost functions for a manufacturing firm are shown in the figure.

a. What is the break-even point?

b. Which region represents profit?



75. One freezer costs \$623.95 and uses 90 kilowatt hours (kwh) of electricity each month. A second freezer costs \$500 and uses 100 kwh of electricity each month. The expected life of each freezer is 12 years. What is the minimum electric rate in cents per kwh for which the 12-year total cost (purchase price + electricity costs) will be less for the first freezer?

exercise 2.5 (cont'd)

43. $x \le \frac{-9 - \sqrt{21}}{2}$ or $x \ge \frac{-9 + \sqrt{21}}{2}$

45. $x \le \frac{1 - \sqrt{33}}{2}$ or $x \ge \frac{1 + \sqrt{33}}{2}$

47. $-1 \le x \le 0$ or $x \ge 1$

49. $x < -1$ or $0 < x < 3$

51. $-2 < x < -1$ or $1 < x < 2$

53. $-2.26 \le x \le 0.76$ or $x \ge 3.51$

55. $0.5 < x < 0.84$ **57.** $x < -\frac{1}{3}$ or $x > 2$

59. $-2 < x < -1$ or $1 < x < 3$

61. $x > 1$ **63.** $x \le -\frac{9}{2}$ or $x > -3$

65. $-3 < x < 1$ or $x \ge 5$

67. $-\sqrt{7} < x < \sqrt{7}$ or $x > 5.34$

69. $x < -3$ or $\frac{1}{2} < x < 5$

71. $x > -1.43$ **73.** $x \le -3.79$ or $x \ge 0.79$

75. Approximately 8.608 cents per kwh

77. More than \$12,500 **79.** Between \$4000 and \$5400

81. $1 < x < 19$ and $y = 20 - x$

83. $10 < x < 35$ **85.** $1 \le t \le 4$ **87.** $2 < t < 2.25$

Section 2.5.A, page 131

1. $-\frac{4}{3} \le x \le 0$ **3.** $\frac{7}{6} < x < \frac{11}{6}$

5. $x < -2$ or $x > -1$ **7.** $x \le -\frac{11}{20}$ or $x \ge -\frac{1}{4}$

9. $x < -\frac{53}{40}$ or $x > -\frac{43}{40}$ **11.** $x \le -\frac{7}{2}$ or $x \ge -\frac{5}{4}$

13. $x < -5$ or $-5 < x < -\frac{4}{3}$ or $x > 6$

15. $-\frac{1}{7} < x < 3$

17. $-\sqrt{3} < x < -1$ or $1 < x < \sqrt{3}$

19. $x < -\sqrt{6}$ or $x > \sqrt{6}$

21. $x \le -2$ or $-1 \le x \le 0$ or $x \ge 1$

23. $0 < x < \frac{2}{3}$ or $2 < x < \frac{8}{3}$

25. $-1.43 < x < 1.24$ **27.** $x < -0.89$ or $x > 1.56$

29. $x \le 2$ or $x \ge \frac{14}{3}$

31. $-1.13 < x < 1.35$ or $1.35 < x < 1.67$

33. If $|x - 3| < \frac{E}{5}$, then multiplying both sides by 5 shows that $5|x - 3| < E$. But $5|x - 3| = |5| \cdot |x - 3| = |5(x - 3)| = |5x - 15| = |(5x - 4) - 11|$. Thus, $|(5x - 4) - 11| < E$.

Chapter 2 Review, page 135

1. $x = 2.7644$ **3.** $x = 3.2678$

5. $x = -3.2843$ **7.** $x = 1.6511$

9. No real solutions **11.** $z = \frac{-3 \pm 2\sqrt{11}}{5}$

13. $x = 3$ or -3 or $\sqrt{2}$ or $-\sqrt{2}$ **15.** 2

17. gold, $\frac{3}{11}$ oz; silver, $\frac{8}{11}$ oz

19. $2\frac{2}{9}$ hrs **21.** 9.6 ft **23.** 4 ft

25. 25 **27.** $(b - 1)^2$ **29.** $x = -\frac{1}{2}$ or $-\frac{11}{2}$

31. $|3x - 1| = 4$

$3x - 1 = 4$ or $3x - 1 = -4$

$3x = 5$ $3x = -3$

$x = \frac{5}{3}$ $x = -1$

33. $\sqrt{x^2 - x - 2} = 0$

Squaring both sides,

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, -1$

Both of these check in the original equation.

35. $\frac{x^2 - 6x + 8}{x - 1} = 0$

Set the numerator equal to 0.

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x = 2, 4$

Neither solution causes the denominator to be 0.

37. $x = \frac{5 - \sqrt{5}}{2}$ **39.** No solutions

41. a. $(-8, \infty)$ **b.** $(-\infty, 5]$ **43.** $\left[\frac{7}{4}, \infty\right)$

45. $(-\infty, -2)$ and $\left(-\frac{1}{3}, \infty\right)$

47. $x \le -1$ or $0 \le x \le 1$ **49.** e

51. $\left[-4, -\frac{5}{8}\right)$ **53.** $x \le -7$ or $x > -4$

55. $x < -2\sqrt{3}$ or $-3 < x < 2\sqrt{3}$

57. $x < \frac{1 - \sqrt{13}}{6}$ or $x > \frac{1 + \sqrt{13}}{6}$

59. $y \le -17$ or $y \ge 13$

61. $x \le -\frac{4}{3}$ or $x \ge 0$