

## LESSON PLAN

Course: Grade 12 U Advanced Functions

Lesson: 6 - 6

Unit/Chapter: Other Function Types

Topic: Local  
Maxima/Minima &  
Concavity

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□ *homework check:* HRW exercise 2.5 p. 124

□ *note:* Local Maxima/Minima and Concavity

Some functions have several areas of increasing or decreasing behaviour. These lead to several peaks and valleys in the graph. Local maxima would represent the highest points in the region or area where they occur. Local minima would represent the lowest points in the region or area where they occur. Calculus is needed to determine exact local maxima or minima, but we can approximate these values by graphing the function or by using a graphing calculator.

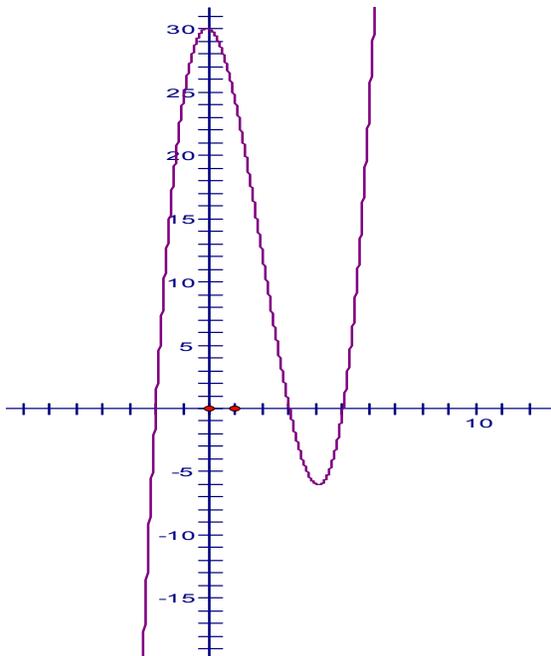
Concavity is used to describe how a curve bends. In most cases, it is easy to determine whether a curve is concave up or down by drawing a line from one point to the other to see if the curve is above or below the line. If the curve is below the line, it is concave up. If the curve is above the line, it is concave down.

Whenever the function changes from concave up to down, there is an inflection point. Again, calculus can be used to determine exactly where the inflection points occur, but we can approximate these values by looking at our graph or with a calculator.

example) use the function  $y = (x + 2)(x - 5)(x - 3)$

Since this function is already factored, the hard part is done. We can make a table of values being sure to include all of the x-intercepts and one more point on either side.

x	y
-4	-126
-3	-48
-2	0
-1	24
0	30
1	24
2	12
3	0
4	-6
5	0
6	24



To find the inflection point, we find the average of the y-values and subtract that average from the max (or add it to the min). If that value does not appear in our graph, we can approximate it using the graph. The average of the y max and y min is 18, therefore, the inflection point occurs at  $y = 12$  which happens to appear in our table. Therefore, the inflection point is  $(2, 12)$  which we can label on our curve. The curve is concave down from  $(-\infty, 2)$  and concave up from  $(2, \infty)$ .

To use the graphing calculator, enter the function in the “y=”

- \* the calculator is automatically set to standard viewing, take a look at the graph

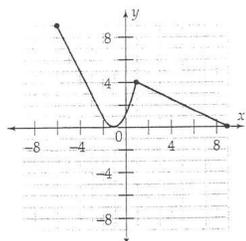
Now make the following changes to get a better look at the local maximum and minimum

- \* change the Xmin to -6 and Xmax to 6
- \* change the Ymin to -40 and the Ymax to 40
- \* use second calc to calculate the maximum setting the left and right bound clear of the maximum point
- \* you get  $x = -0.817$  and  $y = 30.041$
- \* now use second calc to find the minimum in the same way
- \* you get  $x = 4.082$  and  $y = -6.041$
- \* now we can calculate the inflection point the same way we did early by finding the average of the y max and y min.

□ **homework assignment:** HRW exercise 3.2 p. 160 #23 – 28, 33 – 36

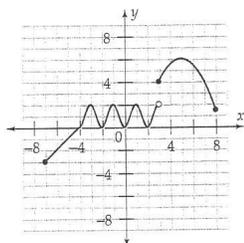
**Exercises 3.2**

In Exercises 1–4, the graph below defines a function,  $f$ . Determine the following:



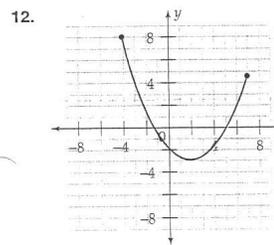
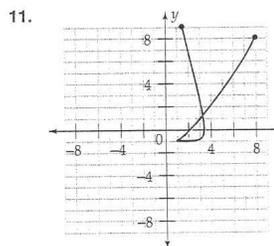
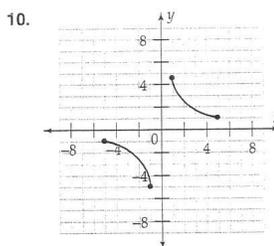
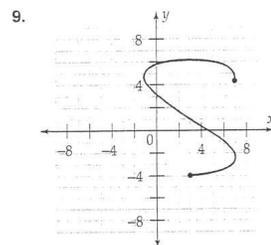
1.  $f(-5)$
2.  $f(1)$
3. the domain of  $f$
4. the range of  $f$

In Exercises 5–8, the graph below defines a function,  $g$ . Determine the following:

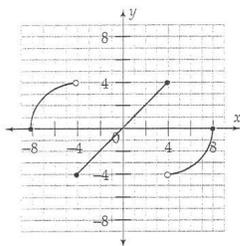


5.  $g(1)$
6.  $g(5)$
7. the domain of  $g$
8. the range of  $g$

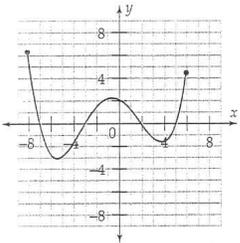
In Exercises 9–14, use the Vertical Line Test to determine whether the graph defines a function. If not, give an example of an input value that corresponds to more than one output value.



13.

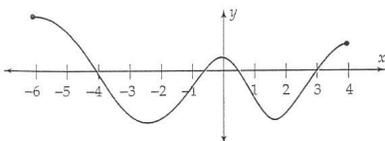


14.

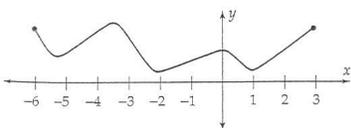


In Exercises 15 and 16, the graph of a function is shown. Find the approximate intervals on which the function is increasing and on which it is decreasing.

15.



16.



In Exercises 17–22, graph each function. Find the approximate intervals on which the function is increasing, decreasing, and constant.

17.  $f(x) = |x - 1| - |x + 1|$

18.  $g(x) = |x - 1| + |x + 2|$

19.  $f(x) = -x^3 - 8x^2 + 8x + 5$

20.  $f(x) = x^4 - 0.7x^3 - 0.6x^2 + 1$

21.  $g(x) = 0.2x^4 - x^3 + x^2 - 2$

22.  $g(x) = x^4 + x^3 - 4x^2 + x - 1$

In Exercises 23–28, graph each function. Estimate all local maxima and minima of the function.

23.  $f(x) = x^3 - x$

24.  $g(t) = -\sqrt{16 - t^2}$

25.  $h(x) = \frac{x}{x^2 + 1}$

26.  $k(x) = x^3 - 3x + 1$

27.  $f(x) = x^3 - 1.8x^2 + x + 2$

28.  $g(x) = 2x^3 + x^2 + 1$

29. a. A rectangle has a perimeter of 100 inches, and one side has length  $x$ . Express the area of the rectangle as a function of  $x$ .

b. Use the function in part a to find the dimensions of the rectangle with perimeter 100 inches and the largest possible area.

30. a. A rectangle has an area of  $240 \text{ in}^2$ , and one side has length  $x$ . Express the perimeter of the rectangle as a function of  $x$ .

b. Use the function in part a to find the dimensions of the rectangle with area  $240 \text{ in}^2$  and the smallest possible perimeter.

31. a. A box with a square base has a volume of  $867 \text{ in}^3$ . Express the surface area of the box as a function of the length  $x$  of a side of the base. (Be sure to include the top of the box.)

b. Use the function in part a to find the dimensions of the box with volume  $867 \text{ in}^3$  and the smallest possible surface area.

32. a. A cylindrical can has a surface area of  $60 \text{ in}^2$ . Express the volume of the can as a function of the radius  $r$ .

b. Use the function in part a to find the radius and height of the can with surface area  $60 \text{ in}^2$  and the largest possible volume.

In Exercises 33–36, graph each function. Find the approximate intervals on which the function is concave up and concave down, and estimate all inflection points.

33.  $f(x) = x^3$

34.  $f(x) = x^3 - 2x$

35.  $h(x) = x^4 - 2x^2$

36.  $g(x) = x^3 - 3x^2 + 2x + 1$

In Exercises 37–40,

- Graph each function.
- Find the approximate intervals on which the function is increasing, decreasing, and constant.
- Estimate all local maxima and minima.
- Find the approximate intervals on which the function is concave up and concave down.
- Estimate the coordinates of any inflection points.

$$37. f(x) = x^2 - 2x + 1 \quad 38. f(x) = -x^2 - 4x - 3$$

$$39. g(x) = x^3 - 3x^2 + 2 \quad 40. g(x) = -x^3 + 4x - 2$$

In Exercises 41–44, sketch the graph of the function. Be sure to indicate which endpoints are included and which are excluded.

$$41. f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } x \geq -1 \end{cases}$$

$$42. g(x) = \begin{cases} |x| & \text{if } x < 1 \\ -3x + 4 & \text{if } x \geq 1 \end{cases}$$

$$43. k(u) = \begin{cases} -2u - 2 & \text{if } u < -3 \\ u - [u] & \text{if } -3 \leq u \leq 1 \\ 2u^2 & \text{if } u > 1 \end{cases}$$

$$44. f(x) = \begin{cases} x^2 & \text{if } x < -2 \\ x & \text{if } -2 \leq x < 4 \\ \sqrt{x} & \text{if } x \geq 4 \end{cases}$$

In Exercises 45–49,

- Use the fact that the absolute-value function is piecewise-defined (see Example 7) to write the rule of the given function as a piecewise-defined function whose rule does not include any absolute value bars.

- Graph the function.

$$45. f(x) = |x| + 2 \quad 46. g(x) = |x| - 4$$

$$47. h(x) = \frac{|x|}{2} - 2 \quad 48. g(x) = |x + 3|$$

$$49. f(x) = |x - 5|$$

In Exercises 50–53, sketch the graph of the function. Be sure to indicate which endpoints are included and which are excluded.

$$50. f(x) = -[x]$$

$$51. g(x) = [-x] \text{ (This is not the same function as in Exercise 50.)}$$

$$52. h(x) = [x] + [-x] \quad 53. f(x) = 2[x]$$

- A common mistake is to graph the function  $f$  in Example 6 by graphing both  $y = x^2$  and  $y = x + 2$  on the same screen, with no restrictions on  $x$ . Explain why this graph could not possibly be the graph of a function.

- Show that the function  $f(x) = |x| + |x - 2|$  is constant on the interval  $[0, 2]$ . *Hint:* Use the piecewise definition of absolute value in Example 7 to compute  $f(x)$  when  $0 \leq x \leq 2$ .

In Exercises 56–59, use your calculator to estimate the domain and range of the function by tracing its graph.

$$56. g(x) = x^2 - 4 \quad 57. h(x) = \sqrt{x^2 - 4}$$

$$58. k(x) = \sqrt{x^2 + 4} \quad 59. f(x) = 3x - 2$$

In Exercises 60 and 61, draw the graph of a function  $f$  that satisfies the given conditions. The function does not need to be given by an algebraic rule.

- $f(-1) = 2$
  - $f(x) \geq 2$  when  $x$  is in the interval  $(-1, \frac{1}{2})$
  - $f(x)$  starts decreasing when  $x = 1$
  - $f(3) = 3 = f(0)$
  - $f(x)$  starts increasing when  $x = 5$
- domain  $f = [-2, 4]$
  - range  $f = [-5, 6]$
  - $f(-1) = f(3)$
  - $f(\frac{1}{2}) = 0$

In Exercises 62–67, graph the curve determined by the parametric equations.

$$62. \begin{cases} x = 0.1t^3 - 0.2t^2 - 2t + 4 \\ y = 1 - t \end{cases} \quad (-5 \leq t \leq 6)$$

$$63. \begin{cases} x = t^2 - 3t + 2 \\ y = 8 - t^3 \end{cases} \quad (-4 \leq t \leq 4)$$

$$64. \begin{cases} x = t^2 - 6t \\ y = \sqrt{t+7} \end{cases} \quad (-5 \leq t \leq 9)$$

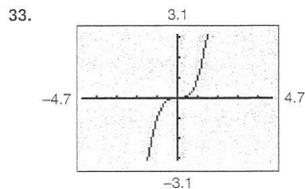
$$65. \begin{cases} x = 1 - t^2 \\ y = t^3 - t - 1 \end{cases} \quad (-4 \leq t \leq 4)$$

$$66. \begin{cases} x = t^2 - t - 1 \\ y = 1 - t - t^2 \end{cases}$$

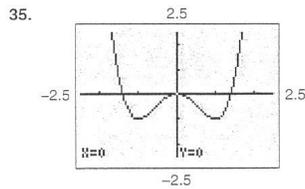
$$67. \begin{cases} x = 3t^2 - 5 \\ y = t^2 \end{cases} \quad (-4 \leq t \leq 4)$$

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1.  $f(-5) = 7$
3. The domain is  $[-6, 9]$ .
5.  $g(1) = 2$
7. The domain is  $[-7, 8]$ .
9. This is not a function; for example, there are three output values for an input value of 4.
11. This is not a function; for example, there are three output values for an input value of 2.
13. This is a function.
15. Increasing on  $(-2.5, 0)$  and  $(1.7, 4)$ ; decreasing on  $(-6, -2.5)$  and  $(0, 1.7)$ .
17. Constant on  $(-\infty, -1]$  and  $[1, \infty)$ ; decreasing on  $(-1, 1)$ .
19. Increasing on  $(-5.8, 0.5)$ ; decreasing on  $(-\infty, -5.8)$  and  $(0.5, \infty)$ .
21. Increasing on  $(0, 0.867)$  and  $(2.883, \infty)$ ; decreasing on  $(-\infty, 0)$  and  $(0.867, 2.883)$ .
23. Minimum at  $x = 0.57735$ ; maximum at  $x = -0.57735$ .
25. Minimum at  $x = -1$ ; maximum at  $x = 1$ .
27. Minimum at  $x = 0.7633$ ; maximum at  $x = 0.4367$ .
29. a.  $A(x) = 50x - x^2$   
b. To maximize area each side should be 25 in. long.
31. a.  $SA(x) = 2x^2 + \frac{3468}{x}$   
b. Base is approximately 9.5354 in.  $\times$  9.5354 in.; height is same (that is, 9.5354 in.).

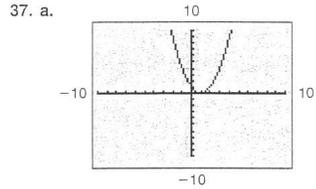


This function is concave up over the interval  $(0, \infty)$  and concave down over the interval  $(-\infty, 0)$ . Therefore, the point of inflection is at  $(0, 0)$ .

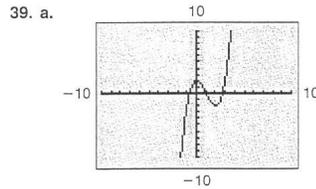


The function is concave up over the approximate intervals  $(-\infty, -0.6)$  and  $(0.6, \infty)$  and concave down over the approximate interval  $(-0.6, 0.6)$ .

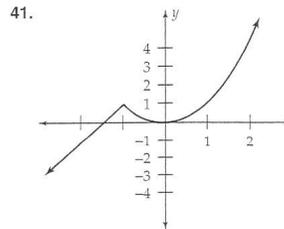
Therefore, the points of inflection are approximately at  $(-0.6, -0.6)$  and  $(0.6, -0.6)$ .



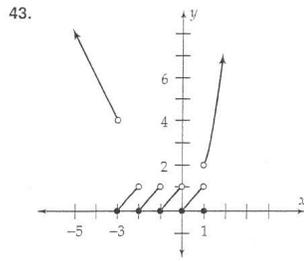
- b. This function is increasing over the interval  $(1, \infty)$  and decreasing over the interval  $(-\infty, 1)$ .
- c. There is a local minimum at the point  $(1, 0)$ .
- d. This function is concave up over the interval  $(-\infty, \infty)$ .
- e. There is no point of inflection.



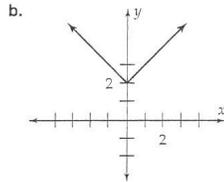
- b. This function is increasing over the intervals  $(-\infty, 0)$  and  $(2, \infty)$  and decreasing over the interval  $(0, 2)$ .
- c. There is a local maximum at the point  $(0, 2)$  and a local minimum at the point  $(2, -2)$ .
- d. This function is concave upward over the interval  $(1, \infty)$  and concave downward over the interval  $(-\infty, 1)$ .
- e. There is a point of inflection at  $(1, 0)$ .



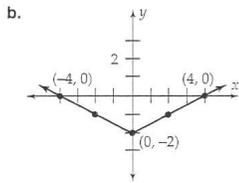
Selected Answers



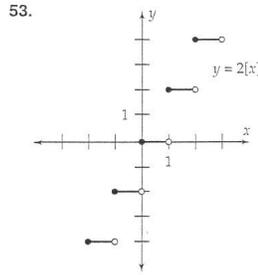
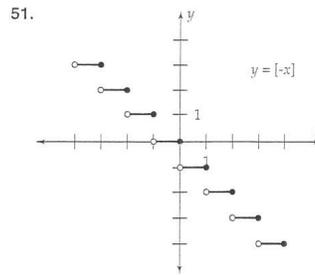
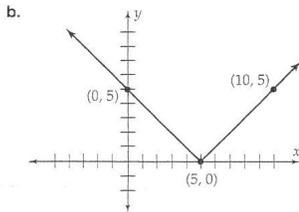
45. a.  $f(x) = \begin{cases} x + 2 & x \geq 0 \\ -x + 2 & x < 0 \end{cases}$



47. a.  $h(x) = \begin{cases} \frac{x}{2} - 2 & x \geq 0 \\ -\frac{x}{2} - 2 & x < 0 \end{cases}$



49. a.  $f(x) = \begin{cases} x - 5 & x \geq 5 \\ 5 - x & x < 5 \end{cases}$



55. When  $0 \leq x \leq 2$ ,  $x$  is positive and  $x - 2$  is negative, so  $|x| = x$  and  $|x - 2| = -(x - 2)$ . Therefore,  $|x| + |x - 2| = x - (x - 2) = x - x + 2 = 2$  when  $0 \leq x \leq 2$ .

57. Domain: all real numbers  $x$  such that  $x \leq -2$  or  $x \geq 2$ ; range: all nonnegative real numbers

59. Domain: all real numbers; range: all real numbers

61. Many correct answers, including

