LESSON PLAN

Course: Grade 12 U Advanced Functions **Day : 85**

Unit/Chapter: Analyzing Change **Topic: Unit Review**

- ***unit review:*** **AW12** self check 1.1 – 1.3 p. 37
  self check 1.4 – 1.6 p. 61
  review exercises p. 62
  self-test p. 67

1. Here is the average airfare for a one-way domestic trip from Ottawa for the years 1988 to 1997.

Year	1988	1989	1990	1991	1992
Air fare ($)	132.50	152.90	164.90	172.60	173.90

Year	1993	1994	1995	1996	1997
Air fare ($)	184.60	202.40	199.00	186.50	179.50

a) Graph the average airfare as a function of time.

b) Calculate the average rate of change of price for each 1-year interval from 1988 to 1997.

c) Over what years are the fares increasing? Is the increase constant? Explain.

2. The Van de Graaff generator at Science North can be charged to several thousand volts to make your hair stand on end! The electric field, E newtons per coulomb, is a function of distance, d metres, from the generator. The equation is $E = \frac{6000}{d^2}$.

a) Graph the function $E = \frac{6000}{d^2}$.

b) Calculate the average rate of change of electric field as a function of distance for each distance interval.

i) 0.5 m to 4.5 m
ii) 1.0 m to 5.0 m
iii) 1.5 m to 5.5 m
iv) 2.0 m to 7.0 m

c) Estimate the instantaneous rate of change of electric field at 2.5 m from the generator.

3. The function $y = x^2 + 3$ is given. Estimate the instantaneous rate of change of y with respect to x at each value of x.

a) 1.0
b) −2.0
c) 2.0

PERFORMANCE ASSESSMENT

4. Use the equation in exercise 2.

a) Estimate the instantaneous rate of change of electric field at 0.75 m. Repeat this calculation at 0.50-m increments up to 4.25 m.

b) Graph the instantaneous rate of change of electric field against distance.

c) Describe the derivative of the electric field function.

d) Compare the graph in part b with the graph in exercise 2a. Describe how each graph shows how the electric field is changing.

1. What is the derivative of each function?

a) $y = x + 1$ b) $y = 2x$ c) $y = 0.5x - 3$

d) $y = x^2 + 4$ e) $y = -2x^3$ f) $y = 5x^3$

2. For each derivative in exercise 1, write a different function that has the same derivative.

3. a) The derivative of $y = x^2$ is $2x$. Use this information to write the derivative of each function.

i) $y = 3x^2$ ii) $y = \frac{1}{2}x^2$ iii) $y = -4x^2$ iv) $y = -3x^2$

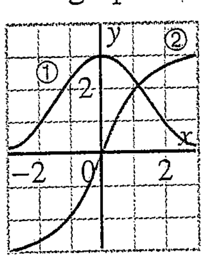
b) What rule did you use in part a?

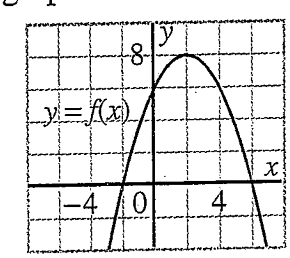
4. a) Sketch the graph of the function $y = |x + 3| - 4$.

b) Sketch the graph of the derivative function.

c) Where is the derivative function in part b not defined? Explain.

5. A function f and its derivative f' are graphed on the same grid (below left). Which graph is the function and which graph is its derivative? Explain.





6. The graph of $y = f(x)$ is given (above right). Sketch the graph of its derivative, $y = f'(x)$.

PERFORMANCE ASSESSMENT

7. a) Calculate the instantaneous rate of change of $y = -x^3$ with respect to x at $x = 2.5$.

b) Determine the equation of the tangent to $y = -x^3$ at $x = 2.5$.

c) There is another point on the graph of $y = -x^3$ at which the instantaneous rate of change of y with respect to x is equal to its value at $x = 2.5$. Determine the equation of the tangent to the curve at this point.

1.1 1. For each data set, sketch the graph. Then calculate the average rates of change of y between each consecutive pair of values of x.

a)

x	0	1	2	3	4
y	3	4	7	12	19

b)

x	0	5	10	15	20
y	0	16	24	28	30

c)

x	−20	−10	0	10	20
y	10	9	7	4	0

2. The salt content of seawater is its *salinity*. This is measured in parts per thousand (ppt). These salinity measurements were recorded in the Bay of Fundy.

Depth, *d* (m)	0.0	5.0	10.0	15.0
Salinity, *S* (ppt)	21.0	28.0	30.0	31.0

Calculate the average rate of change of salinity with depth over each interval.

a) 0.0 m to 15.0 m b) 0.0 m to 5.0 m c) 10.0 m to 15.0 m

1.2 3. The drag, D kilonewtons, experienced by a racing car is a function of the speed, v metres per second, of the car. The equation is $D = 0.08v^2$.

a) Graph the function.

b) Calculate the average rate of change of drag with respect to speed from $v = 20$ m/s to $v = 50$ m/s.

c) Estimate the instantaneous rate of change of drag with respect to speed when $v = 35$ m/s.

4. Calculate the average rate of change of each function between $x = 1$ and $x = 5$.

a) $y = x^2 + x$ b) $y = \sqrt{x+3}$ c) $y = \frac{6}{x}$ d) $y = 7$

5. For each function, estimate the instantaneous rate of change of y with respect to x at $x = -2$ and at $x = 3$.

a) $y = 4x$ b) $y = x^2 + 1$ c) $y = -x^3$ d) $y = 3x + 20$

6. Estimate the instantaneous rate of change of $y = 5x^2$ at $x = 0.5$.

Review Exercises

1.3

7. A car travelling at 50 km/h can stop in about 23 m. A car travelling at 100 km/h needs about 85 m to stop. Here are the data for stopping distances at different speeds.

Speed, v (km/h)	Average stopping distance, d (m)
20	9
30	12
40	16
50	23
60	31
70	42
80	54
90	69
100	85
110	104
120	124
130	147

a) Graph stopping distance against speed.

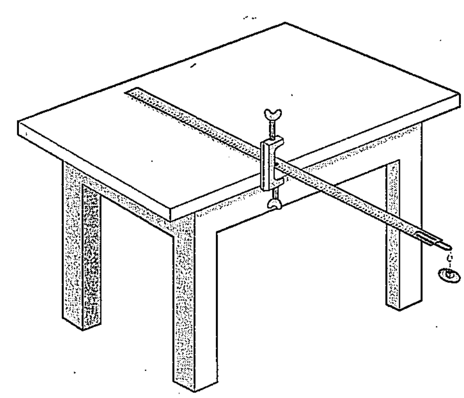
b) The equation $d = 0.01v^2 - 0.25v + 10$ expresses the stopping distance as a function of speed. Use the equation to estimate the instantaneous rate of change of stopping distance for each speed. Use 1-km/h following intervals.

i) 20 km/h
ii) 40 km/h
iii) 60 km/h
iv) 80 km/h
v) 100 km/h

c) Graph the data from part b to make a graph that approximates the derivative function. What special feature does the derivative appear to have?

d) Compare the graphs in parts a and c. Describe how each graph shows how the stopping distance changes.

8. When a person stands at the end of a diving board, the board bends and feels bouncy. This situation can be modelled with a metre stick and some weights. The metre stick is clamped to a table with a length, L centimetres, extended over the edge. The weights are suspended from a paper clip taped to the end of the stick. The downward deflection, d millimetres, of the stick is recorded.



L (cm)	5	10	15	20	25	30	35
d (mm)	0	1	2	3	7	13	21

L (cm)	40	45	50	55	60	65
d (mm)	34	49	68	93	122	157

a) Graph downward deflection against length.

b) Estimate the instantaneous rate of change of deflection every 5 cm, starting at 7.5 cm. Use 5-cm centred intervals.

c) Graph the rate of change of deflection as a function of length.

d) Where is the rate of change of deflection greatest? Use the results of parts b and c to justify your answer.

1.4 9. Determine the derivative of each function.

a) $y = -2$ b) $y = 4x^3$ c) $y = 3 - x^2$ d) $y = 7x + 5$

10. For each function, calculate the value of the derivative at $x = 0.5$.

a) $y = x^2 + 5$ b) $y = -3x + 2$ c) $y = x^4$ d) $y = -2x^2 + 1$

11. Determine the equation of the tangent to the graph of the function $y = -4x^2 + 3$ at $x = 1$.

12. Determine the derivative of each function.

a) $y = \sqrt{x} + 3$ b) $y = -\frac{1}{x}$ c) $y = 2\sin x$ d) $y = \cos x - 4$

13. For each function, determine the equation of the tangent to the graph of the function at $x = 4$.

a) $y = 3\sqrt{x}$ b) $y = \frac{2}{x}$ c) $y = \sqrt{x} - 4$

14. For each function y, determine the value(s) of x where $y' = 2$, if it exists.

a) $y = x^2 + 1$ b) $y = x^3$ c) $y = 4x - 1$

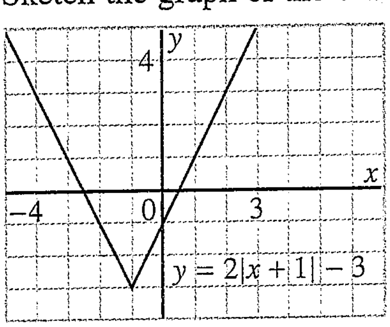
d) $y = 2\sqrt{x}$ e) $y = -\frac{4}{x}$ f) $y = 2x$

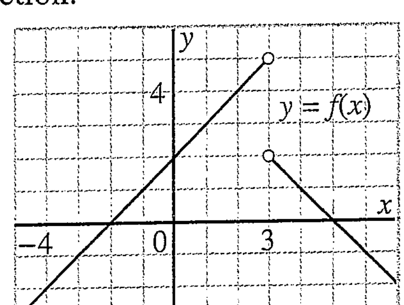
1.5 15. The graph of the function $y = 2|x + 1| - 3$ is shown below left.

a) Describe the graph.

b) Describe the derivative function.

c) Sketch the graph of the derivative function.

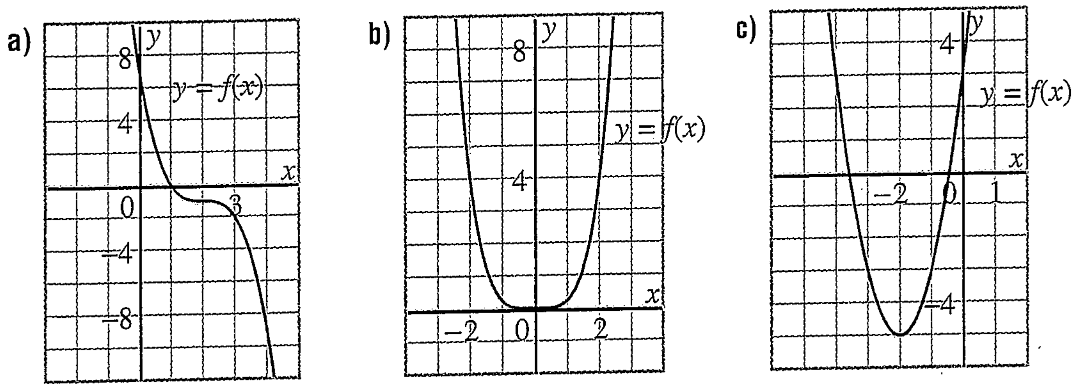


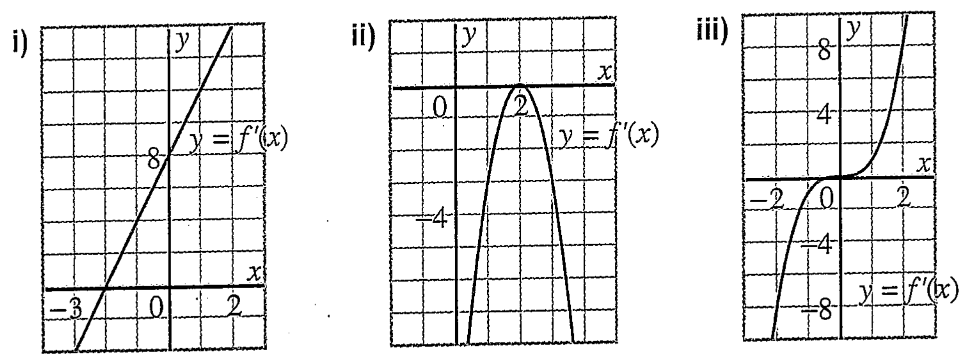


16. Sketch the graph of the derivative of the function above right.

17. The graphs of three functions (parts a to c) and their derivatives (parts i to iii) are shown on the next page. For each function, identify the graph of its derivative. Explain.

Review Exercises

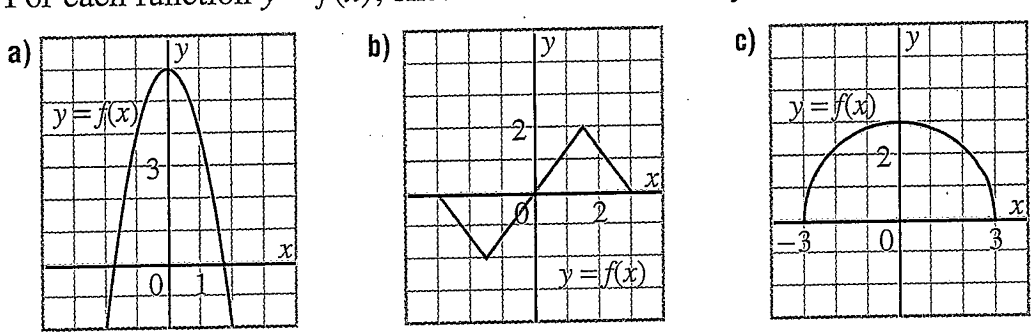




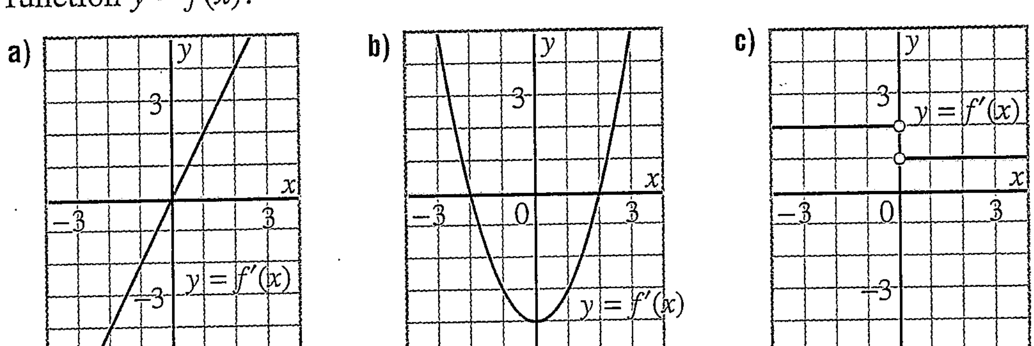
18. List 4 different functions $y = f(x)$ for which $y' = 3$.

19. List 4 different functions $y = f(x)$ for which $y' = 2x$.

20. For each function $y = f(x)$, sketch the derivative of y as a function of x.



21. For each derivative function $y = f'(x)$, sketch a possible graph of the function $y = f(x)$.



1. **Knowledge/Understanding** Statistics Canada has documented the Canadian infant mortality rate for many years. The infant mortality rate is the number of deaths of children up to 1 year of age per 1000 live births.

Year	1960	1965	1970	1975	1980	1985	1990	1995
Infant mortality rate	27.3	23.5	18.7	14.3	10.1	7.8	6.7	6.1

a) Calculate the average rate of change of the mortality rate for each 5-year period.

b) Describe how the average rates of change show that the mortality rate is falling.

c) Describe how the average rates of change show that the mortality rate is levelling off.

d) Explain why the infant mortality rate in Canada has changed substantially since 1960.

2. Consider the function $y = x^2 + 4$.

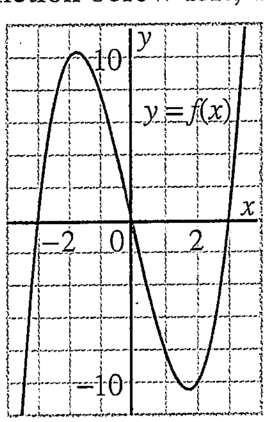
a) Estimate the instantaneous rate of change of y with respect to x at $x = 1$.

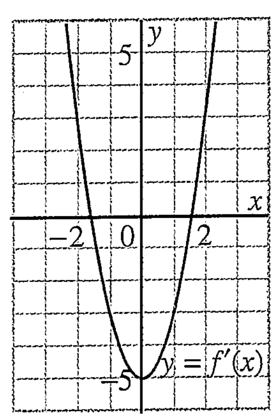
b) Determine the derivative of y with respect to x.

c) Calculate the value of y' at $x = 1$.

d) Explain why your answers for parts a and c should be the same.

3. **Communication** Explain how to sketch the graph of the derivative of the function below left; then sketch the graph.





4. The graph of the derivative of a function is shown above right. Sketch a possible graph of the function.

Self-Test

5. Consider the function $y = \sqrt{x}$.

   a) Calculate the average rate of change of y with respect to x over each interval. Give the answers to 2 decimal places where necessary.

      i) $x = 0$ to $x = 10$
      ii) $x = 0$ to $x = 1$
      iii) $x = 0$ to $x = 0.1$
      iv) $x = 0$ to $x = 0.01$

   b) Explain why the instantaneous rate of change does not exist at $x = 0$.

6. **Application** A wood stove operates efficiently at flue temperatures between 150°C and 200°C (423K and 473K). Its energy output is given by the function $E = \frac{T^4}{5\,000\,000} - 1600$, where E is the energy output in watts and T is the Kelvin temperature.

   a) Determine the average rate of change of energy output with temperature between 423K and 473K.

   b) Estimate the instantaneous rate of change of energy output at 450K.

7. Determine the equation of the tangent to the graph of $y = -2x^2 + 4$ at the point (1, 2).

8. **Thinking/Inquiry/Problem Solving** Consider the function $y = \frac{1}{x} + 2$.

   a) Graph the function for x between –5 and 5.

   b) Describe the graph of the function for values of x greater than 5 and less than –5.

   c) Sketch the graph of the derivative y'.

   d) Use the behaviour of y' for large values of x to explain your answer in part b.

PERFORMANCE ASSESSMENT

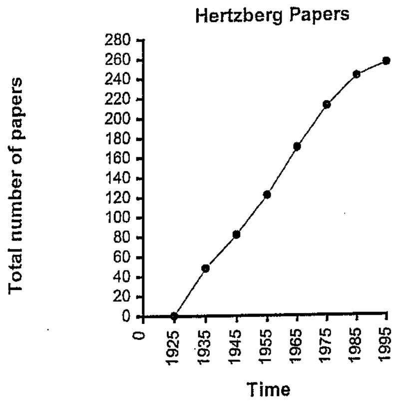
9. The cross section of a hill is parabolic with equation $y = -0.1x^2 + 4$.

   a) Describe what x and y represent.

   b) Explain why the steepness of the hill at a point on the hill is given by the derivative of y with respect to x.

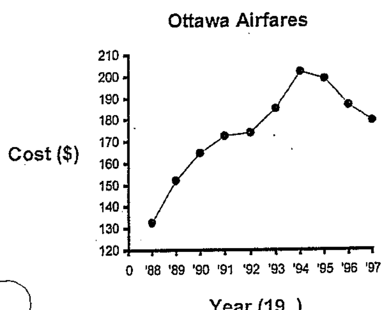
   c) A ramp is placed on the hill so that one end of the ramp is a tangent to the hill at the point (–4, 2.4). Determine the equation of the ramp.

   d) Determine the coordinates of the point where the other end of the ramp meets the level ground.



Self-Check 1.1–1.3, page 37

1. a)

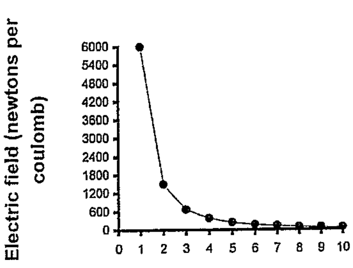


b)

Year	Avera[illegible]ge of [illegible])
1988–1989	20.4[illegible]
1989–1990	12[illegible]
1990–1991	[illegible]
1991–1992	[illegible]
1992–1993	1[illegible]
1993–[illegible]	17.80
19[illegible]	–3.40
[illegible]–1996	–12.50
[illegible]1997	–7.00

c) The far[illegible]m 1988 to 1994. No, the increase is [illegible]stant.

2. a)

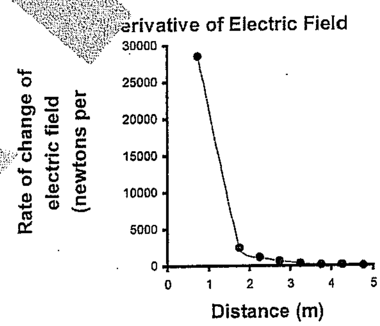


b) **i)** –5926 newtons per coulomb/m
ii) –1440 newtons per coulomb/m
iii) –617 newtons per coulomb/m
iv) –276 newtons per coulomb/m
c) –768 newtons per coulomb/m

3. a) 2 **b)** –4 **c)** 4

4. a) Estimates may vary. The estimates below were calculated using a 0.0001-m interval.

Distance (m)	Instantaneous rate of change of electric field ([illegible]s/coulomb/m)
0.75	–28 439
1.25	[illegible]143
1.75	[illegible]
2.25	–[illegible]
2.75	–57[illegible]
[illegible].25	–350
3.75	–228
[illegible]	–156

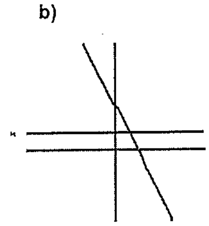


1.4 Exercises, page 43

1. a) $y' = 3$ **b)** $y' = -0.5$ **c)** $y' = -1$
d) $y' = 1$ **e)** $y' = 2x$ **f)** $y' = 3x^2$

2. a) The graph of f is a linear function with a positive slope.
b) The graph of f is a linear function with a negative slope.

3. a) $y' = -2$
b)



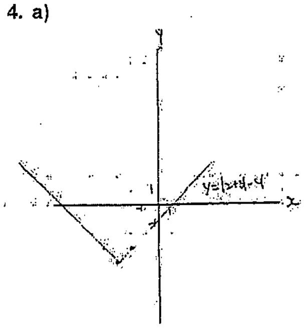
c) Answers may vary. $y = -2x + 5$

1. a) $y' = 1$ b) $y' = 2$ c) $y' = 0.5$
d) $y' = 2x$ e) $y' = -6x^2$ f) $y' = 15x^2$

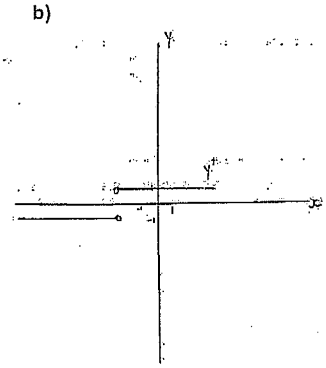
2. Answers may vary. Sample answers follow:
a) $y = x - 3$ b) $y = 2x + 4$ c) $y = 0.5x$
d) $y = x^2 - 1$ e) $y = -2x^3 + 5$ f) $y = 5x^3 - 2$

3. a) i) $y' = 6x$ ii) $y' = x$ iii) $y' = -8x$ iv) $y' = -6x$
b) Vertical stretch rule

4. a)



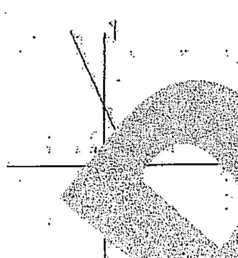
b)



c) At $x = -3$

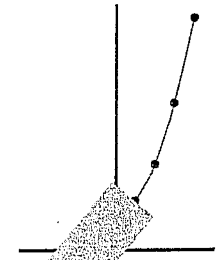
5. ② is f; ① is f'.

6.

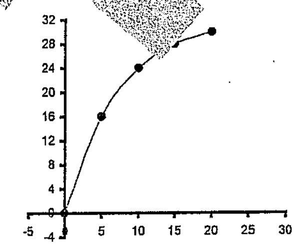


7. a) -18.75 b) $y = -18.75x + 31.25$
c) $y = -18.75x - 31.25$

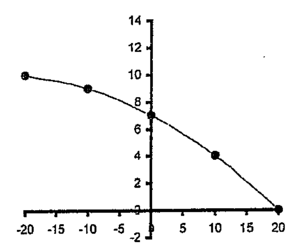
1. a) 1; 3; 5; 7



b) 3.2; 1.6

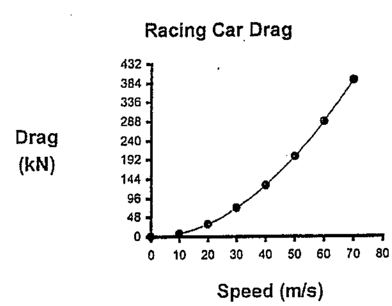


c) −0.1; −0.2; −0.3; −0.4



2. a) 0.67 ppt/m b) 1.4 ppt/m c) 0.2 ppt/m

3. a)



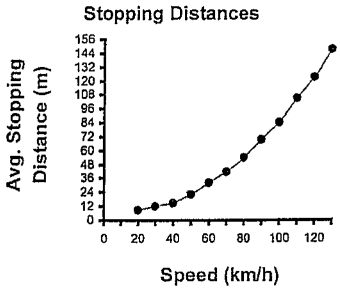
b) 5.6 kN/m/s c) 5.6 kN/m/s

4. a) 7 b) 0.21 c) −1.2 d) 0

5. Estimates may vary.
a) 4; 4 b) −4; 6 c) −12; −27 d) 3; 3

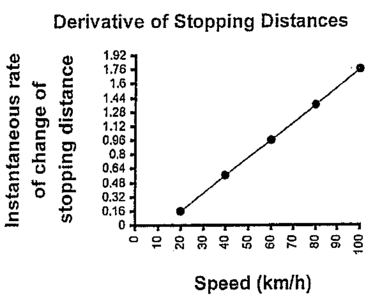
6. Estimates may vary. 5

7. a)

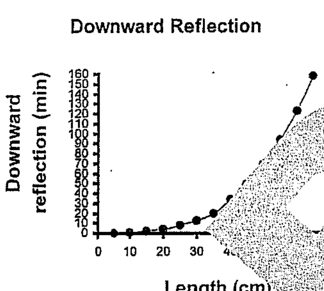


b) i) 0.16 m/km/h ii) 0.56 m/km/h iii) 0.96 m/km/h
iv) 1.36 m/km/h v) 1.76 m/km/h

c) The graph of the derivative function appears to be linear.



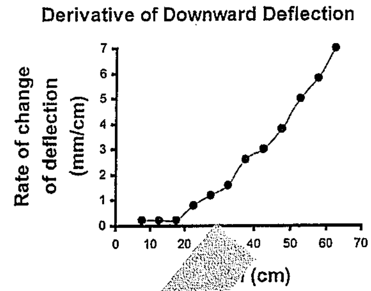
8. a)



b)

Length (cm)	[illegible]imated [illegible]us rate o[illegible] of de[illegible]n (m[illegible]n)
7.5	[illegible]2
12.5	0.2
17.5	0.2
22.5	0.8
27.5	1.2
32.5	1.6
37.5	2.6
42.5	3.0
47.5	3.8
52.5	5.0
57.5	5.8
62.5	7.0

b)



c) The rate [illegible] of deflection is greatest between lengths [illegible] cm.

9. a) $y' = 0$ b) $y' =$ [illegible] c) $y' = -2x$ d) $y' = 7$

10. a) 1 b) -3 c) 0.5 d) -2

11. [illegible] $8x + 7$

12. a) $y' = \frac{1}{[illegible]x}$ b) $y' = \frac{1}{x^2}$

c) [illegible] $2\cos x$ d) $y' = -\sin x$

13. a) [illegible] 3

b) $y = -\frac{1}{8}x + 1$

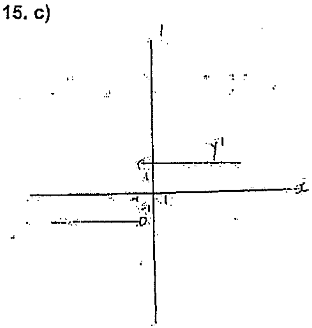
c) $y = \frac{1}{4}x - 3$

14. a) 1 b) $\pm\sqrt{\frac{2}{3}}$ c) No values

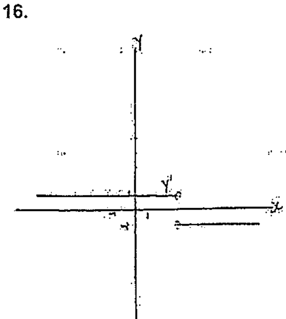
d) $\frac{1}{4}$ e) $\pm\sqrt{2}$

f) All real values

15. c)



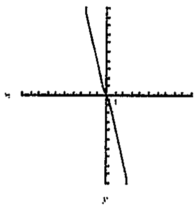
16.



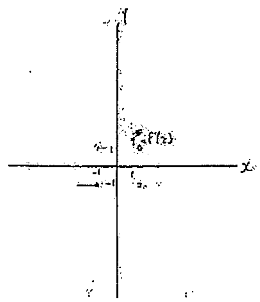
17. a) ii) b) iii c) i

18. Answers may vary. $y = 3x$, $y = 3x + 1$, $y = 3x - 1$, $y = 3x - 2$

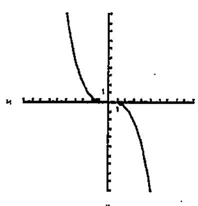
19. Answers may vary. $y = x^2$, $y = x^2 + 1$, $y = x^2 - 1$, $y = x^2 - 2$



b)

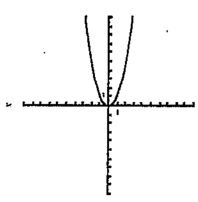


c)

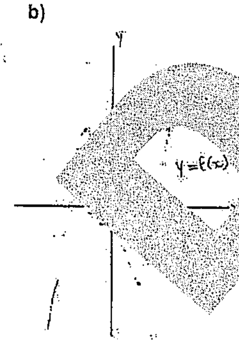


21. Graphs may vary.

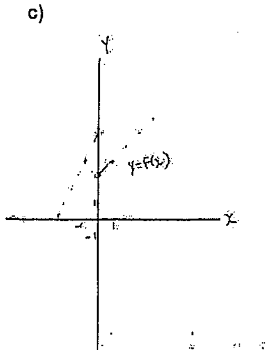
a)



b)

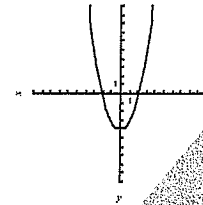


c)

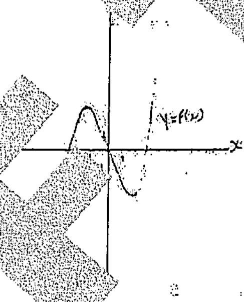


Self-Test, page 67

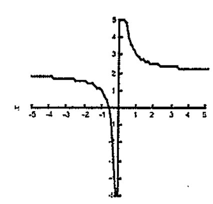
1. a) –0.76, –0.96, –0.88, –0.84, –0.46, –0.22, –0.12
2. a) 2 b) $y' = 2x$ c) $y' = 2$
3.



4. Graphs may vary.

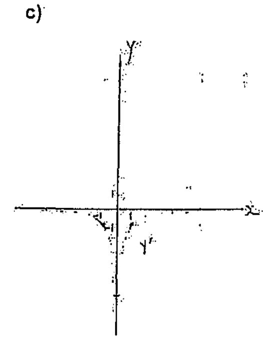


5. a) i) 0.316 ii) 1 iii) 3.162 iv) 10
   b) The function $y = \sqrt{x}$ has a vertical tangent at $x = 0$.
6. a) 72.2 W/K b) 72.9 W/K
7. $y = -4x + 6$
8. a)



   b) For values of x greater than 5, the y-values get closer and closer to 2, but never reach 2. For values of x less than –5, the y-values get closer and closer to 2 but never reach 2.

   c)



9. c) $x = 0.8x + 5.6$
   d) (–7, 0)