## Lesson Plan

Grade 10 Academic Math
Unit: Polynomials

Lesson: 3

## Topic: $\quad$ Special Products

## homework check: FM 10 p. 62 \# 2-4

## \# note: Special Products

Knowing of special products is important for both expanding and factoring. For example, expand the difference of squares given.
a) $(x+5)(x-5)=$
$=x^{2}-5 x+5 x-25$
$=x^{2}-25$
Notice that because the factors have the same terms separated by opposite sign, they 'zero' each other out. Adding larger coefficients or terms does not change the pattern. For example,
b) $(2 x-3)(2 x+3)=$
$=4 x^{2}+6 x-6 x-9$
$=4 x^{2}-9$

Therefore, the pattern for expanding the factors of a difference of squares $(a-b)(a+b)$ yields the polynomial $a^{2}-b^{2}$. Expand the following using the pattern.
c) $(4 x-5)(4 x+5)=$
$=16 x^{2}-25$

We can establish a pattern for expanding a sum of squares in the same way.
d) $(x+4)^{2}=(x+4)(x+4)$
$=x^{2}+4 x+4 x+16$
$=x^{2}+8 x+16$
Notice that because the factors are exactly the same, the middle term is twice as big. Adding larger coefficients or terms does not change the pattern. For example,
e) $(3 x-4)^{2}=(3 x-4)(3 x-4)$
$=9 x^{2}-12 x-12 x+16$
$=9 x^{2}-24 x+16$
Therefore, the pattern for expanding the factors of a sum of squares $(a+b)^{2}=(a+b)(a+b)$ yields the polynomial $a^{2}+2 a b+b^{2}$. Expand the following using the pattern.
f) $(2 x-3)^{2}=(2 x-3)(2 x-3)$
$=4 x^{2}-12 x+9$
These patterns allow the expansions to be done quickly without distributive property and collection of like terms, however it does require a bit of memorization.
\# homework assignment: FM 10 p. 65 \# 1-5

## EXERCISE 2.4

B 1. Expand.
(a) $(x+3)^{2}$
(b) $(x-2)^{2}$
(c) $(x+5)^{2}$
(d) $(x-4)(x+4)$
(e) $(y+2)^{2}$
(f) $(m-7)^{2}$
(g) $(t+5)(t-5)$
(h) $(x+6)(x-6)$
(i) $(y+1)^{2}$
(j) $(x-9)^{2}$
(k) $(x+10)^{2}$
(I) $(x-7)(x+7)$
(m) $(x+12)^{2}$
(n) $(x-6)^{2}$
(0) $(y-1)(y+1)$
(p) $(a+b)(a-b)$
3. Evaluate the following using one of the special products $(a+b)^{2},(a-b)^{2}$, or $(a+b)(a-b)$.
(a) $52^{2}$
(b) $(40-5)(40+5)$
(c) $95^{2}$
(d) $71^{2}$
(e) $101^{2}$
(f) $(30+3)(30-3)$
(g) $83^{2}$
(h) $(4+50)(4-50)$
4. Expand and simplify.
(a) $2 x+(x+4)^{2}$
(b) $x^{2}-(2 x-1)^{2}$
(c) $3 a b+(2 a-7 b)^{2}$
(d) $-4 x^{2}+(3 x-y)(3 x+y)$
(e) $(2 a-3 b)^{2}-\left(a^{2}+4 a b\right)$
(f) $(a b-2 c)(a b+2 c)-(a b)^{2}$
5. Expand and simplify.
(a) $(x+2)^{2}+(x-5)^{2}$
(b) $2(x-4)(x+4)-x^{2}$
(c) $(x+5)(x-3)+(x+4)^{2}$
(d) $7-(x-6)^{2}$
(e) $(3+2 x)(3-2 x)-(3+2 x)^{2}$
(f) $4(5 x-1)^{2}-2(3 x+1)(3 x-1)$
2. Expand and simplify.
(a) $(3 x-5)(3 x+5)$
(b) $(2 x+7)^{2}$
(c) $(4 x+5)^{2}$
(d) $(2 x-3 y)^{2}$
(e) $(5 x-y)(5 x+y)$
(f) $(a+2 b)^{2}$
(g) $(a b+2)(a b-2)$
(h) $\left(x^{2}-4\right)^{2}$
(i) $(4-3 m)(4+3 m)$
(j) $(6+7 x)^{2}$


We know that $(a+b)^{2}=a^{2}+2 a b+b^{2}$
Using the algorithm for long multiplication of polynomials, we can obtain the following expansions for the powers of $(a+b)$.

$$
\begin{aligned}
(a+b)^{3}= & (a+b)^{2}(a+b) \\
= & a^{3}+3 a^{2} b+3 a b^{2}+b^{3} \\
(a+b)^{4}= & (a+b)^{3}(a+b) \\
= & a^{4}+4 a^{3} b+6 a^{2} b^{2}+4 a b^{3} \\
& +b^{4}
\end{aligned}
$$

The coefficients obtained in the various expansions of $(a+b)^{n}$ where $n=0,1$, 2,3 , and 4, can be arranged in a triangular form called Pascal's triangle.

$$
\begin{aligned}
1 & \leftarrow(\mathrm{a}+\mathrm{b})^{0} \\
& =1
\end{aligned}
$$

## EXERCISE 2.4

1. (a) $x^{2}+6 x+9$
(e) $y^{2}+4 y+4$
(i) $y^{2}+2 y+1$
(m) $x^{2}+24 x+144$
2. (a) $9 x^{2}-25$
(e) $25 x^{2}-y^{2}$
(i) $16-9 \mathrm{~m}^{2}$
3. (a) 2704
(e) 10201
4. (a) $x^{2}+10 x+16$
(d) $5 x^{2}-y^{2}$
5. (a) $2 x^{2}-6 x+29$
(d) $-x^{2}+12 x-29$
6. (a) $5+2 \sqrt{6}$
(e) $22+12 \sqrt{2}$
(i) 18
7. (a) 8091
(b) $x^{2}-4 x+4$
(f) $m^{2}-14 m+49$
(j) $x^{2}-18 x+81$
(n) $x^{2}-12 x+36$
(b) $4 x^{2}+28 x+49$
(f) $a^{2}+4 a b+4 b^{2}$
(j) $36+84 x+49 x^{2}$
(b) 1575
(f) 891
(b) $-3 x^{2}+4 x-1$
(e) $3 a^{2}-16 a b+9 b^{2}$
(b) $x^{2}-32$
(e) $-8 x^{2}-12 x$
(b) $7-2 \sqrt{10}$
(c) $9-6 \sqrt{2}$
(f) $278-160 \sqrt{3}$
(j) 8
(b) 9975
(c) $x^{2}+10 x+25$
(g) $\mathrm{t}^{2}-25$
(k) $x^{2}+20 x+100$
(o) $y^{2}-1$
(c) $16 x^{2}+40 x+25$
(g) $a^{2} b^{2}-4$
(c) 9025
(g) 6889
(g) 1
$(k)-5$
(d) $x^{2}-16$
(h) $x^{2}-36$
(I) $x^{2}-49$
(p) $a^{2}-b^{2}$
(d) $4 x^{2}-12 x y+9 y^{2}$
(h) $x^{4}-8 x^{2}+16$
(d) 5041
(h) -2484
(c) $4 a^{2}-25 a b+49 b^{2}$
(f) $-4 c^{2}$
(c) $2 x^{2}+10 x+1$
(f) $82 x^{2}-40 x+6$
(d) $13-2 \sqrt{42}$
(h) -1
(l) 28
(d) 1479
