

Lesson Plan

Grade 10 Academic Math

Lesson: 2 - 5

Unit: Linear Systems

Topic: Solving Word Problems using Linear Systems

✚ **homework check:** Principles of Mathematics 10 p. 54 # 4, 6, 11

✚ **note:** Solving Word Problems using Linear Systems

Many word problems can be represented and solved using linear systems. In order to be successful in this, we need to know a couple of things.

First: All unknowns must be represented by a variable which involves writing ‘let’ statements.

Second: We need to know that the speed of a vehicle is the distance it travels divided by the time it takes to get there. (If you forget this remember that speed is represented in the units of km/h.) This same formula can be rearranged in several ways to represent the other two variables. So time is distance divided by speed and distance is speed multiplied by time.

Third: All percentages must be represented in decimal form. When solving a mixing solutions question, be sure to represent any percentages in decimal form.

Fourth: The speed of a boat or plane travelling “with the current/wind” is represented by adding the speeds together. The speed of a boat or plane “against the wind/current” is represented by subtracting the speeds.

examples)

- a) A boat travelled 288 miles downstream and back. The trip downstream took 12 hours. The trip back took 16 hours. Find the speed of the boat in still water and the speed of the current.

Solution:

Step 1: Write “let” statements

Let “b” represent the speed of the boat in still water.

Let “c” represent the speed of the current.

Step 2: Identify the important information you know.

Total Distance: 288km

Speed downstream: $b - c$ taking 12 h

Speed upstream: $b + c$ taking 16 h

Step 3: Write your equations knowing that distance = speed X time.

$$16b - 16c = 288 \quad (1)$$

$$12b + 12c = 288 \quad (2)$$

Step 4: Solve the system using the appropriate process.

$$16b - 16c = 288 \quad (1)$$

$$12b + 12c = 288 \quad (2)$$

$$(1) \times 12 \quad \text{and} \quad (2) \times 16$$

$$192b - 192c = 3456$$

$$192b + 192c = 4608$$

$$384b = 8064$$

$$b = 21$$

$$12(21) + 12c = 288$$

$$252 + 12c = 288$$

$$12c = 36$$

$$c = 3$$

Therefore, the boat travels at 21 mph and the current is 3 mph.

- b) Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge, you decide to mix 10% solution with 30% solution to make your own 15% solution. You need 10 liters of the 15% acid solution. How many liters of 10% solution and 30% solution should you use?

Step 1: Write your “let” statements.

Let x represent the number of litres of 10% solution.

Let y represent the number of litres of 30% solution.

Step 2: Identify the important information.

There are two solutions that need to produce a 15% mixture.

The mixture must be 10L total.

Weight Equation:

$$x + y = 10 \quad (1)$$

Mixture Equation:

$$0.1x + 0.3y = 1.5 \quad (2)$$

Step 3: Solve the system using the appropriate process.

$$x + y = 10 \quad (1)$$

$$0.1x + 0.3y = 1.5 \quad (2)$$

$$(1) - (2) \times 10$$

$$x + y = 10$$

$$-[x + 3y = 15]$$

$$-2y = -5$$

$$y = 2.5$$

$$x + 2.5 = 10$$

$$x = 7.5$$

Therefore, you need 7.5 L of 10% solution and 2.5 L of 30% solution.

- c) Jessie is driving 1420 km. He drives part way at 125 km/h and part way at 105 km/h. If the whole trip takes 12 hours, how far did he drive at each speed?

Step 1: Write your “let” statements

Let x represent the distance driven at 125 km/h

Let y represent the distance driven at 105 km/h

Step 2: Identify important information.

Distance Total: 1420 km

Distance Equation:

$$x + y = 1420 \quad (1)$$

Time total: 12 hours

Time Equation (remember that time = distance/speed):

$$\frac{x}{125} + \frac{y}{105} = 12 \quad (2)$$

Step 3: Solve the problem using the appropriate process.

$$x + y = 1420 \quad (1)$$

$$\frac{x}{125} + \frac{y}{105} = 12 \quad (2)$$

(1) $\times 125$ and (2) $\times 13125$ to get rid of fraction

$$125x + 125y = 177500$$

$$105x + 125y = 157500$$

subtract

$$20x = 20000$$

$$x = 1000$$

$$1000 + y = 1420$$

$$y = 420$$

Therefore, Jessie drove 1000km at 125 km/h and drove 420km at 105 km/h.

✚ homework assignment: Principles of Mathematics 10 p. 27 # 6, 7, 8, 9, 10, 11, 12, 13