Course: MFM2P Gr. 10 Applied **Lesson: 7 - 5**

Unit: Representing Quadratic Relations **Topic: Solving Problems Involving Quadratics**

- ***homework check:*** **Lesson 7 - 4**

- ***note:*** **Solving Problems Involving Quadratics**

In order to solve problems that involve quadratics, we rely on our skills for finding x and y intercepts, and max or min values. For example,

a) A golfer hits a tee shot into the rough and the ball stops approximately 120 yds from the green. There is a tree located 40 yds from the ball, directly in the path of the shot. The golfer decides to try to hit the ball over the tree. The path of the shot can be modeled by the equation $h = -0.01875x^2 + 2.25x$, where h is the height of the ball and x is the horizontal distance in yards from where the second shot is taken.

i) How tall must the tree be to stop the ball?
This tree is located 40 yds from the ball. Because x represents the horizontal distance, we can substitute x = 40 into the equation.

$h = -0.01875x^2 + 2.25x$

$h = -0.01875(40)^2 + 2.25(40)$

$h = -30 + 90$

$h = 60\,yds$

The tree must be 60 yds tall (almost 60 metres).

ii) Does the golfer hit the green with the shot?
To find this information, we factor to find the x intercepts.

$h = -0.01875x^2 + 2.25x$

not a trinomial, so we common factor ONLY

$h = -0.1875x(x - 120)$

$x = 0$ or 120

Therefore, the ball begins at 0 and lands at 120 yards. The shot is successful!

iii) What is the maximum height of the ball and when does it occur?
We know where the x intercepts occur. We use this information to find the middle value of x which tells us when the max occurs.

$$\frac{0+120}{2} = 60$$

use this value to find the max height

$y = -0.01875x^2 + 2.25x$

$y = -0.01875(60)^2 + 2.25(60)$

$y = -67.5 + 135$

$y = 67.5 yds$

The max height of 67.5 yds occurs when the ball is 60 yds away.

b) A diver dives off a dock into the water to find a coral reef. The swim path can be modeled by the equation $d = 0.2x^2 - 3.6x + 6.4$ where d is the depth in metres of the diver and x is the horizontal distance in metres from the diver's point of entry.

i) If an observer sees a shark surfacing 5 m from the point of entry. Is the diver safe from the shark?

Because x represents the distance from the point of entry, substitute x = 5.

$d = 0.2x^2 - 3.6x + 6.4$

$d = 0.2(5)^2 - 3.6(5) + 6.4$

$d = 0.1 - 7.2 + 6.4$

$d = -0.7$

No the diver is not safe at only -0.7 m below the surface.

ii) What is the horizontal distance the diver swims?

We need to know the x intercepts to gauge the distance between point of entry and surfacing.

$d = 0.2x^2 - 3.6x + 6.4$

$d = 0.2(x^2 - 18x + 32)$

$d = 0.2(x - 16)(x - 2)$

$x = 2$ and 16

The diver goes into the water 2 m from the dock and surfaces another 16 m from the dock.

Therefore, he swims a horizontal distance of 14 m.

iii) How deep is the coral reef?

We need to know the minimum value. Since we already know the x intercepts, find the middle and substitute.

$$\frac{16+2}{2} = 9$$

$$d = 0.2x^2 - 3.6x + 6.4$$

$$d = 0.2(9)^2 - 3.6(9) + 6.4$$

$$d = 16.2 - 32.4 + 6.4$$

$$d = -9.8m$$

The reef is -9.8 m under the surface.

- ***homework assignment:*** **Lesson 7 - 5**

Lesson 7 – 5: Solving Problems Relying on Quadratics

1. The school council is planning Battle of the Bands. They expect that 700 tickets will be sold if the ticket price is $5. A survey shows that if the ticket price is raised by $0.50, there will be 50 fewer tickets sold. This situation can be modelled by the equation $R = -25p^2 + 100p + 3500$, where R represents the revenue from the ticket sales in dollars and p represents the number of times the ticket price is increased by $0.50.

a) Find the maximum revenue the student council can make.

b) What is the price of a ticket to get the maximum revenue?

c) What is the original revenue before the ticket price is raised?

d) Which portion of the quadratic relation represents the original revenue?

2. Two free kicks are taken in a soccer match. Each kick is directed at the goal, and the goalkeeper has no chance of making either save. The crossbar of the goal is 2.44m high. The path of each kick can be modelled by a quadratic relation, where h represents the height of the ball in metres and d represents the horizontal distance in metres.

Kick #1: $h = -0.007d^2 + 0.28d$

Kick #2: $h = -0.007d^2 + 0.25d$

a) Could each kick score if the free kicks are taken from 25m?

b) Could each kick score if they are taken from 20m?

3. A set of fireworks is designed to explode at the highest point. The equations that model the pathways of the four different types of fireworks are

A: $h = -5t^2 + 50t$

B: $h = -5t^2 + 60t$

C: $h = -5t^2 + 70t$

D: $h = -5t^2 + 80t$

a) Find the height and time when each firework explodes.

b) Which firework explodes at the highest point?

c) Which pathway is the narrowest? How wide is the path?

d) Which firework path is the widest? How wide is the path?

4. A bus company services 240 000 riders per day at a fare of $2. The city wants to raise the fare to cover expenses but a survey indicates that for every $0.10 increase in price, riders will drop by 10 000 people. The equation $R = -1000p^2 + 4000p + 480000$ **, where R is the revenue in dollars and p is the number of times the bus fare is collected shows the relationship.**

a) What is the maximum revenue of the new bus fare?

b) How much is the increase in bus fare? What is the new bus far?

c) How many people will be riding the bus?