Teacher: **Mrs. Roen** *Unit:* **Geometry**

Course: **MAP 4C** *Lesson:* **2 – 4 Optimizing Area and Perimeter**

- ***Homework:*** **2-3**
- ***Note:*** **Optimizing Area and Perimeter**

Among all rectangles with a given perimeter, a **square** has the maximum area. Likewise, among all rectangles with a given area, a **square** has the minimum perimeter. Sometimes constraints are added to the optimization problem in which a square is not the best shape. This happens when one side of the rectangle already exists as perhaps the rectangle is built beside a house. In these cases, it is best to use tables and graphs to find the dimensions of the optimal rectangle. Remember, in a graph, a maximum area would be the highest point on the parabola while the minimum perimeter would be the lowest point on the parabola.

For example, a rectangular garden is to be fenced using the wall of a shed as the fourth side of the rectangle. The garden is to have an area of 48m^2. Determine the dimensions of the garden that will minimize the perimeter if whole numbers are required.

In this case, the perimeter of the garden can be found using a new formula $P = 2L + w$ because the second width will be the side of the shed and will not be included in fencing. We can make a table that reflects all the length and widths that yield an area of 48m^2 and find the corresponding perimeters.

Length (m)	Width (m)	Perimeter (m)
48	1	97
24	2	50
16	3	35
12	4	28
8	6	22
6	8	20
4	12	20
3	16	22
2	24	28
1	48	50

From our table there are two possible rectangles that yield a minimum perimeter of 20 meters. The client could pick either or the preferred shapes.

If we were allowed to use decimals for our dimensions, we could look at possible numbers that still yield an area of 48m^2 in between a length of 6m and 4 m to get a smaller perimeter.

The shape that maximizes area and minimizes perimeter in a rectangle is a square. The formulas that help us with the application are $A = s^2$ and $P = 4s$. We use these formulas as follows…

a) Given an area of 65m^2 find the minimum perimeter.

Solution:

$$A = s^2$$
$$65 = s^2$$
$$\sqrt{65} = s$$
$$s = 8.1m$$

then we calculate the minimum perimeter using

$$P = 4s$$
$$P = 4(8.1)$$
$$P = 32.4m$$

b) Given the perimeter of 80cm find the maximum area.

Solution:

$$P = 4s$$
$$80 = 4s$$
$$\frac{80}{4} = s$$
$$s = 20cm$$

then we can calculate the maximum area using

$$A = s^2$$
$$A = 20^2$$
$$A = 400cm^2$$

Of course, a rectangle is not the only shape that can be maximized or minimized. A circle or triangle can also be maximized. In this case, we are usually given an area and asked to find the radius or given a circumference (perimeter) and asked to find the radius.

- ***Homework:*** **2 – 4 Optimizing Area and Perimeter**

Lesson: **<u>2 – 4 Optimizing Area and Perimeter</u>**

1. For each perimeter, what are the dimensions of the rectangle with the maximum area? What is the area? **(2 marks each)**

a) 40 cm

b) 110 feet

c) 25 m

d) 87 mm

2. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is that perimeter? **(2 marks each)**

a) 20 ft^2

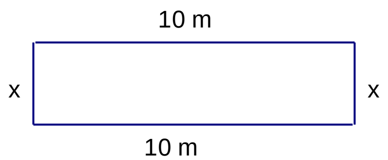
b) 81 m^2

c) 144 cm^2

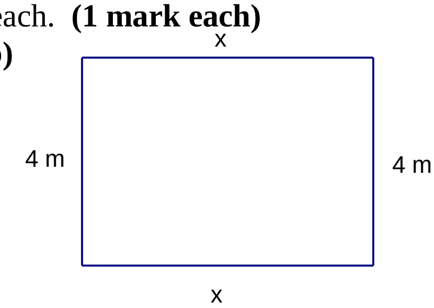
d) 60 in^2

3. A gardener uses 24 m of fencing to enclose a rectangular garden. Some possible rectangles are shown. Determine the missing dimension for each. **(1 mark each)**

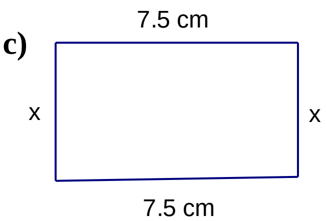
a)



b)



c)



4. A farmer has 650 feet of fencing. Does he have enough fencing to enclose a rectangular area of half an acre? Show your work. *1 acre is equal to 45 560 square feet.*

(4)

5. A lifeguard is roping off a rectangular swimming area using the beach as one side. She has 200 m of rope.
a) Determine the greatest area she can rope off and its dimensions.

(3)

b) Is the area greater of less than 50 000 square feet? Show your work. *1 foot equals 0.3048 m*

(2)

6. Josie buys 20 m of dog fencing to create a dog pen. How much more area will the dog have if Josie builds a circular pen rather than a square on? Show your work.

(5)